Solution Keys to MAT3210 Assignment 7

1.Solution

(1) .

$$\min\ z = 50u_1 + 10u_2 + 2u_3$$

subject to

$$\begin{aligned} u_1 &\geq 0 \\ u_2 &\geq 0 \\ u_3 &\geq 0 \\ 3u_1 + u_2 + u_3 &\geq 40 \\ -3u_1 - u_3 &\geq 20 \\ 5u_1 + u_2 + 4u_3 &= 2 \end{aligned}$$

(2) .

$$\min\ z = 5u_1 + 2u_2 + 20u_3$$

subject to

$$\begin{aligned} u_1 &\geq 0 \\ u_2 &\geq 0 \\ u_3 &\geq 0 \\ u_1 + u_2 + 7u_3 &= 3 \\ 2u_1 + u_2 + 3u_3 &= 1 \\ -u_2 - 5u_3 &= 0 \end{aligned}$$

(3) .

$$\min\ z = 5u_1 + 3u_2$$

subject to

$$u_1 \;\; free$$

$$u_2 \leq 0$$

$$u_1 - u_2 = 5$$

$$2u_1 + 5u_2 \leq 6$$

(5) .

$$\max \; z = 5u_1 + 6u_2$$

subject to

$$u_1 \leq 0$$

$$u_2 \geq 0$$

$$2u_1 + 3u_2 = 1$$

$$u_1 - u_2 = 1$$

(6) .

$$\min \; z = b_1 u_1 + b_2 u_2 + b_3 u_3$$

subject to

$$u_1 \leq 0$$

$$u_2 \;\; free$$

$$u_3 \geq 0$$

$$a_{11}u_1 + a_{21}u_2 + a_{31}u_3 \geq c_1$$

$$a_{12}u_1 + a_{22}u_2 + a_{32}u_3 \leq c_2$$

$$a_{13}u_1 + a_{23}u_2 + a_{33}u_3 = c_3$$

(7) .

We express the primal problem:

$$\max\ x_0 = c^T x$$

subject to

$$\begin{aligned} Ax &= b \\ x_i \geq l_i\ \ i &= 1\ ,\ \cdots\ ,\ n \\ x_i \leq u_i\ \ i &= 1\ ,\ \cdots\ ,\ n \\ x_i\ \ free\ \ i &= 1\ ,\ \cdots\ ,\ n \end{aligned}$$

Then the dual problem is :

$$\min\ x_0 = \sum_{i=1}^{m} b_i v_i + \sum_{i=1}^{n} l_i v_{i+m} + \sum_{i=1}^{n} u_i v_{i+m+n}$$

subject to

$$\begin{aligned} v_i\ \ free\ \ for\ i = 1\ ,\ \cdots\ ,\ m \\ v_i \leq 0\ \ for\ i = m+1\ ,\ \cdots\ ,\ m+n \\ v_i \geq 0\ \ for\ i = m+n+1\ ,\ \cdots\ ,\ m+2n \\ \sum_{i=1}^{m} a_{ij} v_i + v_{j+m} + v_{j+m+n} = c_j\ for\ j = 1\ ,\ \cdots\ ,\ n \end{aligned}$$

where $A = (a_{ij})$