Solution Keys to MAT3210 Assignment 6

1.Solution

Transform the problem into the standard form and use simplex methods:

Tableau 1 :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	3	-3	5	1	0	0	50
x_5	1	0	1	0	1	0	10
x_6	1*	-1	4	0	0	1	2
x_0	-40	-20	-2	0	0	0	0

$\overrightarrow{y_2} \leq 0$, then the solution space in x_2 direction is unbound . And $z_2 - c_2 < 0$, then optimal value is $+\infty$.

2.Solution

Transform the problem into the standard form :

Initial Tableau :

	x_1	x_2	x_3	x_4	b
	3	2	-1	0	6
	2	1	0	1	2
x_0	-2	-5	0	0	0

Use the M-Method , consider a new LPP' :

$$\max\ z = 2x_1 + 5x_2 - Mx_5$$

subject to

$$3x_1 + 2x_2 - x_3 + x_5 = 6$$

$$2x_1 + x_2 + x_4 = 2$$

$$x_1 \ , \ x_2 \ , \ ; x_3 \ , \ x_4 \ , \ x_5 \geqslant 0$$

Initial Tableau' :

	x_1	x_2	x_3	x_4	x_5	b
	3	2	-1	0	1	6
	2	1	0	1	0	2
x_0	-2	-5	0	0	M	0

Tableau 1 :

	x_1	x_2	x_3	x_4	x_5	b
x_5	3	2	-1	0	1	6
x_4	2^*	1	0	1	0	2
x_0	$-2-3M$	$-5-2M$	M	0	0	$-6M$

Tableau 2 :

	x_1	x_2	x_3	x_4	x_5	b
x_5	0	$\frac{1}{2}$	-1	$-\frac{3}{2}$	1	3
x_1	1	$\frac{1}{2}^*$	0	$\frac{1}{2}$	0	1
x_0	0	$-4-\frac{1}{2}M$	M	$1+\frac{3}{2}M$	0	$2-3M$

Tableau 3 :

	x_1	x_2	x_3	x_4	x_5	b
x_5	-1	0	-1	-2	1	2
x_2	2	1	0	1	0	2
x_0	$8+M$	0	M	$5+2M$	0	$10-2M$

We see that this is the final table , but the artificial variable in M-Method is still basic at positive level . We conclude that the feasible solution is empty .

The proof of the conclusion:

If the feasible region is not empty ,we have at least one point $(x_1^0\ ,\ x_2^0)$ in the feasible region . Then $(x_1^0\ ,\ x_2^0\ ,\ 0\ ,\ 0\ ,\ 0)$ is a feasible solution of the LPP' in the M-Method , and the objective value at this point is $2x_1^0+5x_2^0$, and the optimal value of the LPP' through simplex method is $10-2M$ since we choose M large enough $2x_1^0+5x_2^0 > 10-2M$, then it is contradict to the fact that $10-2M$ is the optimal value .

Use the Two Phase-Method , Phase 1:

consider a new LPP" :

$$\max\ z = -x_5$$

subject to

$$3x_1 + 2x_2 - x_3 + x_5 = 6$$

$$2x_1 + x_2 + x_4 = 2$$

$$x_1\ ,\ x_2\ ,\ ;x_3\ ,\ x_4\ ,\ x_5 \geqslant 0$$

Initial Tableau' :

	x_1	x_2	x_3	x_4	x_5	b
	3	2	-1	0	1	6
	2	1	0	1	0	2
x_0	0	0	0	0	1	0

Tableau 1 :

	x_1	x_2	x_3	x_4	x_5	b
x_5	3	2	-1	0	1	6
x_4	2^*	1	0	1	0	2
x_0	-3	-2	1	0	0	-6

Tableau 2 :

	x_1	x_2	x_3	x_4	x_5	b
x_5	0	$\frac{1}{2}$	-1	$-\frac{3}{2}$	1	3
x_1	1	$\frac{1}{2}^*$	0	$\frac{1}{2}$	0	1
x_0	0	$-\frac{1}{2}$	1	$\frac{3}{2}$	0	-3

Tableau 3 :

	x_1	x_2	x_3	x_4	x_5	b
x_5	-1	0	-1	-2	1	2
x_2	2	1	0	1	0	2
x_0	1	0	1	2	0	-2

We see that this is the final table , but the artificial variable is still basic at positive level . We conclude that the feasible solution of the original LPP is empty .

The proof of the conclusion:

If the feasible region is not empty ,we have at least one point $(x_1^0 \ , \ x_2^0)$ in the feasible region . Then $(x_1^0 \ , \ x_2^0 \ , \ 0 \ , \ 0 \ , \ 0)$ is a feasible solution of the LPP" and the objective value at this point is 0 , but the optimal value of the LPP" through simplex method is -2 , then we get contradiction .

3.Solution

we eliminate x_1 :

Initial Tableau :

x_1	x_2	x_3	x_4	x_5	b
1	1	1	0	0	1
1	2	3	-1	0	3
-1	1	-2	0	1	1
-1	0	1	0	0	0

Equivalent problem :

x_2	x_3	x_4	x_5	b
1	2	-1	0	2
2	-1	0	1	2
1	2	0	0	1

Use the M-Method:

Tableau 1 :

	x_2	x_3	x_4	x_5	x_6	b
x_6	1	2	-1	0	1	2
x_5	2	-1	0	1	0	2
x_0	1	2	0	0	M	1

Tableau 2 :

	x_2	x_3	x_4	x_5	x_6	b
x_6	1	2^*	-1	0	1	2
x_5	2	-1	0	1	0	2
x_0	$1-M$	$2-2M$	M	0	0	$1-2M$

Tableau 3 :

	x_2	x_3	x_4	x_5	b
x_3	$\frac{1}{2}$	1	$-\frac{1}{2}$	0	1
x_5	$\frac{5}{2}$	0	$-\frac{1}{2}$	1	3
x_0	0	0	1	0	-1

We find that $\overrightarrow{y_3} \leq 0$,then the feasible region is unbound . The optimal solution is not unique since $z_1 - c_1 = 0$ and $\min\{\frac{x_{Bi}}{y_{i1}} | y_{i1} > 0\} = \frac{6}{5} > 0$. Set $x_2 = 1 < \frac{6}{5}$, then $(1\ ,\ \frac{1}{2}\ ,\ 0\ ,\ \frac{1}{2})$ is a nonbasic optimal solution .

Finally we get the maximizer -1 with a solution (0 , 0 , 1 , 0 , 3) of the original problem .

4.Solution

(a) The feasible region is unbounded in direction x_6 since $\overrightarrow{y_6} \leq 0$. (b) The optimal solution is not unique since $z_2 - c_2 = 0$ and $\min\{\frac{x_{Bi}}{y_{i2}} | y_{i2} > 0\} = \frac{2}{3} > 0$. Set $x_2 = \frac{1}{2} < \frac{2}{3}$, then $(\frac{1}{2}\ ,\ \frac{1}{2}\ ,\ 5\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ \frac{1}{2})$ is a nonbasic optimal solution .

5.Solution

Use simplex method to solve the problem:

Initial Tableau :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	1	2	0	1	0	0	5
x_5	1^*	1	-1	0	1	0	2
x_6	7	3	-5	0	0	1	20
x_0	-3	-1	0	0	0	0	0

Tableau 1 :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	0	1	1^*	1	-1	0	3
x_1	1	1	-1	0	1	0	2
x_6	0	-4	2	0	-7	1	6
x_0	0	2	-3	0	3	0	6

Tableau 2 :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_3	0	1	1	1	-1	0	3
x_1	1	2	0	1	0	0	5
x_6	0	-6	0	2	-5	1	0
x_0	0	5	0	3	0	0	15

It is the final table and we get a degenerate optimal solution $(5\ ,\ 0\ ,\ 3\ ,\ 0\ ,\ 0\ ,\ 0)$. For $\overrightarrow{y_5} \leq 0$,$z_5 - c_5 = 0$ we know $(5\ ,\ 0\ ,\ 3 + \Theta\ ,\ 0\ ,\ \Theta\ ,\ 5\Theta)$ is a nonbasic feasible solution for all positive Θ .

6.Solution

Transform the problem into standard form , and use two phase method:

Phase 1:

Initial Tableau :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	2	1	1	1	0	0	2
x_6	3	4	2	0	-1	1	8
x_0	0	0	0	0	0	1	0

Tableau 1 :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	2	1^*	1	1	0	0	2
x_6	3	4	2	0	-1	1	8
x_0	-3	-4	-2	0	1	0	-8

Tableau 2 :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	2	1	1	1	0	0	2
x_6	-5	0	-2	-4	-1	1	0
x_0	5	0	2	4	1	0	0

We see that the artificial variable x_6 is still basic at the end of Phase 1 .

Phase 2 :

Initial Tableau :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	2	1	1	1	0	0	2
x_6	-5	0	-2	-4	-1	1	0
x_0	-3	-2	-3	0	0	0	0

Tableau 1 :

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	2	1	1^*	1	0	0	2
x_6	-5	0	-2^{**}	-4	-1	1	0
x_0	1	0	-1	2	0	0	4

We can't use 1^* as the pivot element or we get the artificial variable positive , we choose -2^{**} as the pivot element . We drop x_6 column when it is nonbasic .

Tableau 2 :

	x_1	x_2	x_3	x_4	x_5	b
x_2	$-\frac{1}{2}$	1	0	-1	$-\frac{1}{2}$	2
x_3	$\frac{5}{2}$	0	1	2	$\frac{1}{2}$	0
x_0	$\frac{7}{2}$	0	0	4	$\frac{1}{2}$	4

Finally we get the maximizer 4 with optimal solution (0 , 2 , 0 , 0 , 0) .

7.Solution

(a) True

(b) False : If at the end of Phase 1 or two -phase method we have one or more y_i basic at zero level , we also get a degenerate **FS**

(c) True

(d) False : If $z_j - c_j \leq 0$ for all j and $z_i - c_i = 0$ for some i but not all $y_{ji} \leq 0$ and $\min\{\frac{x_{Bj}}{y_{ji}} | y_{ji} > 0\} = 0$ then the optimal solution is also unique .

(e) False

(f) False

(g) True