Solution Keys to MAT 3210 Assignment 5

1. Solution

Standard form:

$$\begin{aligned} \text{maximize} \quad & z = -4x_1 - x_2 \\ \text{subject to} \quad & 3x_1 + x_2 = 3 \\ & 4x_1 + 3x_2 - x_3 = 6 \\ & x_1 + 2x_2 + x_4 = 4 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Introducing artificial variables x_5, x_6, x_7, we get

◇ Tableau 1

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	**b**
x_5	3	1	0	0	1	0	0	3
x_6	4	3	-1	0	0	1	0	6
x_7	1	2	0	1	0	0	1	4
x_0	4	1	0	0	M	M	M	0

◇ Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	**b**
x_5	3^*	1	0	0	1	0	0	3
x_6	4	3	-1	0	0	1	0	6
x_7	1	2	0	1	0	0	1	4
x_0	$4-8M$	$1-6M$	M	$-M$	0	0	0	$-13M$

◇ Tableau 3

	x_1	x_2	x_3	x_4	x_6	x_7	**b**
x_1	1	$\frac{1}{3}$	0	0	0	0	1
x_6	0	$\frac{5}{3}^*$	-1	0	1	0	2
x_7	0	$\frac{5}{3}$	0	1	0	1	3
x_0	0	$-\frac{1+10M}{3}$	M	$-M$	0	0	$-4-5M$

◇ Tableau 4

	x_1	x_2	x_3	x_4	x_7	**b**
x_1	1	0	$\frac{1}{5}$	0	0	$\frac{3}{5}$
x_2	0	1	-$\frac{3}{5}$	0	0	$\frac{6}{5}$
x_7	0	0	1^*	1	1	1
x_0	0	0	$-\frac{1+5M}{5}$	$-M$	0	$-\frac{18+5M}{5}$

◇ Tableau 5: Initial tableau and also the optimal tableau.

	x_1	x_2	x_3	x_4	**b**
x_1	1	0	0	-$\frac{1}{5}$	$\frac{2}{5}$
x_2	0	1	0	$\frac{3}{5}$	$\frac{9}{5}$
x_3	0	0	1	1	1
x_0	0	0	0	$\frac{1}{5}$	$-\frac{17}{5}$

2. Solution

(a) Standard form

$$\begin{aligned} \text{maximize} \quad & z = 2x_1 + 3x_2 - 5x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 7 \\ & 2x_1 - 5x_2 + x_3 - x_4 = 10 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

Introducing artificial variables x_5, x_6, we get

◇ Tableau 1

	x_1	x_2	x_3	x_4	x_5	x_6	**b**
x_5	1	1	1	0	1	0	7
x_6	2	-5	1	-1	0	1	10
x_0	-2	-3	5	0	M	M	0

◇ Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	**b**
x_5	1	1	1	0	1	0	7
x_6	2^*	-5	1	-1	0	1	10
x_0	$-2-3M$	$-3+4M$	$5-2M$	M	0	0	$-17M$

◇ Tableau 3

	x_1	x_2	x_3	x_4	x_5	**b**
x_5	0	$\frac{7}{2}^*$	$\frac{1}{2}$	$\frac{1}{2}$	1	2
x_1	1	-$\frac{5}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	0	5
x_0	0	$-\frac{16+7M}{2}$	$\frac{12-M}{2}$	$-\frac{2+M}{2}$	0	$10-2M$

◇ Tableau 4: Initial and also the optimal tableau.

	x_1	x_2	x_3	x_4	**b**
x_2	0	1	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{4}{7}$
x_1	1	0	$\frac{6}{7}$	-$\frac{1}{7}$	$\frac{45}{7}$
x_0	0	0	$\frac{50}{7}$	$\frac{1}{7}$	$\frac{102}{7}$

(b) Standard form

$$\begin{aligned} \text{maximize} \quad & z = -4x_1 + 8x_2 - 3x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 7 \\ & 2x_1 - 5x_2 + x_3 - x_4 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

Introducing artificial variables x_5, x_6, we get

◇ Tableau 1

	x_1	x_2	x_3	x_4	x_5	x_6	**b**
x_5	1	1	1	0	1	0	7
x_6	2	-5	1	-1	0	1	10
x_0	4	-8	3	0	M	M	0

◇ Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	**b**
x_5	1	1	1	0	1	0	7
x_6	2^*	-5	1	-1	0	1	10
x_0	$4-3M$	$-8+4M$	$3-2M$	M	0	0	$-17M$

◇ Tableau 3

	x_1	x_2	x_3	x_4	x_5	**b**
x_5	0	$\frac{7}{2}^*$	$\frac{1}{2}$	$\frac{1}{2}$	1	2
x_1	1	-$\frac{5}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	0	5
x_0	0	$\frac{4-7M}{2}$	$\frac{2-M}{2}$	$\frac{4-M}{2}$	0	$-20-2M$

◇ Tableau 4: Initial and also the optimal tableau.

	x_1	x_2	x_3	x_4	**b**
x_2	0	1	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{4}{7}$
x_1	1	0	$\frac{6}{7}$	-$\frac{1}{7}$	$\frac{45}{7}$
x_0	0	0	$\frac{5}{7}$	$\frac{12}{7}$	$-\frac{148}{7}$

3. Solution

Standard form:

$$\begin{aligned} \text{maximize} \quad & x_0 = 4x_1 + 5x2 - 3x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 10 \\ & x_1 - x_2 + x_4 = 1 \\ & 2x_1 + 3x_2 + x_3 + x_5 = 20 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

*(i)*Two-Phase method

Phase I

◇ Tableau 0: Initial tableau

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	**b**
1	1	1	0	0	1	0	0	10
1	-1	0	-1	0	0	1	0	1
2	3	1	0	1	0	0	1	20
0	0	0	0	0	1	1	1	0

◇ Tableau 1

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	**b**
y_1	1	1	1	0	0	1	0	0	10
y_2	1^*	-1	0	-1	0	0	1	0	1
y_3	2	3	1	0	1	0	0	1	20
x_0	-4	-3	-2	1	-1	0	0	0	-31

◇ Tableau 2

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	**b**
y_1	0	2	1	1	0	1	-1	0	9
x_1	1	-1	0	-1	0	0	1	0	1
y_3	0	5^*	1	2	1	0	-2	1	18
x_0	0	-7	-2	-3	-1	0	4	0	-27

◇ Tableau 3

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	**b**
y_1	0	0	$\frac{3}{5}^*$	$\frac{1}{5}$	$-\frac{2}{5}$	1	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{9}{5}$
x_1	1	0	$\frac{1}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{23}{5}$
x_2	0	1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	0	$-\frac{2}{5}$	$\frac{1}{5}$	$\frac{18}{5}$
x_0	0	0	$-\frac{3}{5}$	$-\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{6}{5}$	$\frac{7}{5}$	$-\frac{9}{5}$

◇ Tableau 4: Final tableau of Phase I

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	**b**
x_3	0	0	1	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	3
x_1	1	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	4
x_2	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	3
x_0	0	0	0	0	0	1	$-\frac{7}{5}$	1	0

Phase II

◇ Tableau 0: Initial tableau

	x_1	x_2	x_3	x_4	x_5	**b**
x_3	0	0	1	$\frac{1}{3}$	$-\frac{2}{3}$	3
x_1	1	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	4
x_2	0	1^*	0	$\frac{1}{3}$	$\frac{1}{3}$	3
x_0	-4	-5	3	0	0	0

◇ Tableau 1

	x_1	x_2	x_3	x_4	x_5	**b**
x_3	0	0	1	$\frac{1}{3}$	$-\frac{2}{3}$	3
x_1	1^*	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	4
x_2	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	3
x_0	-4	0	3	$\frac{5}{3}$	$\frac{5}{3}$	15

◇ Tableau 2

	x_1	x_2	x_3	x_4	x_5	**b**
x_3	0	0	1	$\frac{1}{3}^*$	$-\frac{2}{3}$	3
x_1	1	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	4
x_2	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	3
x_0	0	0	3	-1	3	31

◇ Tableau 3: Optimal tableau

	x_1	x_2	x_3	x_4	x_5	**b**
x_4	0	0	3	1	-2	9
x_1	1	0	2	0	-1	10
x_2	0	1	-1	0	1	0
x_0	0	0	11	0	1	40

*(ii)*M-method

◇ Tableau 0: Initial tableau

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	**b**
x_6	1	1	1	0	0	1	0	0	10
x_7	1	-1	0	-1	0	0	1	0	1
x_8	2	3	1	0	1	0	0	1	20
x_0	-4	-5	3	0	0	M	M	M	0

◇ Tableau 1

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	**b**
x_6	1	1	1	0	0	1	0	0	10
x_7	1^*	-1	0	-1	0	0	1	0	1
x_8	2	3	1	0	1	0	0	1	20
x_0	$-4-4M$	$-5-3M$	$3-2M$	M	$-M$	0	0	0	$-31M$

◇ Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	x_8	**b**
x_6	0	2	1	1	0	1	0	9
x_1	1	-1	0	-1	0	0	0	1
x_8	0	5^*	1	2	1	0	1	18
x_0	0	$-9-7M$	$3-2M$	$-4-3M$	$-M$	0	0	$4-27M$

◇ Tableau 3

	x_1	x_2	x_3	x_4	x_5	x_6	**b**
x_6	0	0	$\frac{3}{5}^*$	$\frac{1}{5}$	$-\frac{2}{5}$	1	$\frac{9}{5}$
x_1	1	0	$\frac{1}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{23}{5}$
x_2	0	1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{18}{5}$
x_0	0	0	$\frac{24-3M}{5}$	$-\frac{2+M}{5}$	$\frac{9+2M}{5}$	0	$\frac{182-9M}{5}$

◇ Tableau 4

	x_1	x_2	x_3	x_4	x_5	**b**
x_3	0	0	1	$\frac{1}{3}^*$	$-\frac{2}{3}$	3
x_1	1	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	4
x_2	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	3
x_0	0	0	0	-2	5	22

◇ Tableau 5: Optimal tableau

	x_1	x_2	x_3	x_4	x_5	**b**
x_4	0	0	3	1	-2	9
x_1	1	0	2	0	-1	10
x_2	0	1	-1	0	1	0
x_0	0	0	6	0	1	40

4. Solution

$$\text{maximize } x_0 = x_4 + x_1 + x_2$$

Eliminate x_4. Choose the first row.

x_1	x_2	x_3	x_4	**b**
2	2	0	1^*	7
4	3	-1	1	11
3	3	0	1	13
-1	-1	0	-1	0

→

x_1	x_2	x_3	x_4	**b**
2	2	0	1	7
2	1	-1	0	4
1	1	0	0	6
1	1	0	0	7

Phase I

◇ Tableau 0: Initial tableau

x_1	x_2	x_3	y_1	y_2	**b**
2	1	-1	1	0	4
1	1	0	0	1	6
0	0	0	1	1	0

◇ Tableau 1

	x_1	x_2	x_3	y_1	y_2	**b**
y_1	2^*	1	-1	1	0	4
y_2	1	0	0	0	1	6
x_0	-3	-2	1	0	0	-10

◇ Tableau 2

	x_1	x_2	x_3	y_1	y_2	**b**
x_1	1	$\frac{1}{2}^*$	$-\frac{1}{2}$	$\frac{1}{2}$	0	2
y_2	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	4
x_0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	0	-4

◇ Tableau 3

	x_1	x_2	x_3	y_1	y_2	**b**
x_2	2	1	-1	1	0	4
y_2	-1	0	1^*	-1	1	2
x_0	1	0	-1	2	0	-2

◇ Tableau 4: Final tableau of Phase I

	x_1	x_2	x_3	y_1	y_2	**b**
x_2	1	1	0	1	1	6
x_3	-1	0	1	-1	1	2
x_0	0	0	0	1	1	0

Phase II

◇ Tableau 0: Initial tableau and also the optimal tableau.

	x_1	x_2	x_3	**b**
x_2	1	1	0	6
x_3	-1	0	1	2
x_0	1	1	0	7

$$x_4 = 7 - 2x_1 - 2x_2 = -5.$$

Optimal solution: $x^* = [0, 6, 2, -5]^T$, $x_0^* = 1$.

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