MAT3210 Assignment 4 Suggested Solution

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Q.1) Solution:

Basic	x_1	x_2	x_3	x_4	Solution
x_3	1	1*	1	0	2
x_4	1	-1	0	1	1
z	1	-2	0	0	0

$\longrightarrow$

Basic	x_1	x_2	x_3	x_4	Solution
x_2	1	1	1	0	2
x_4	2	0	1	1	3
z	1	0	2	0	4

So the optimal solution is $(0,2,0,3)^T$, with the optimal value 4. □

Q.2) Solution:

Basic	x_1	x_2	x_3	x_4	x_5	Solution
x_3	1	1	1	0	0	2
x_4	0	1	0	1	0	1
x_5	1*	-1	0	0	1	1
z	-2	1	0	0	0	0

$\longrightarrow$

Basic	x_1	x_2	x_3	x_4	x_5	Solution
x_3	0	2*	1	0	-1	1
x_4	0	1	0	1	0	1
x_1	1	-1	0	0	1	1
z	0	-1	0	0	2	2

$\longrightarrow$

Basic	x_1	x_2	x_3	x_4	x_5	Solution
x_2	0	1	1/2	0	-1/2	1/2
x_4	0	0	-1/2	1	1/2	1/2
x_1	1	0	1/2	0	1/2	3/2
z	0	0	1/2	0	3/2	5/2

So the optimal solution is $(3/2,1/2,0,1/2,0)^T$, with optimal value 5/2. □

Q.3) Solution:

I	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
x_5	1	2	2	4	1	0	0	40
x_6	2	-1	1*	2	0	1	0	8
x_7	4	-2	1	-1	0	0	1	10
z	-3	1	-3	-4	0	0	0	0

$\longrightarrow$

II	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
x_5	-3	4*	0	0	1	-2	0	24
x_6	2	-1	1	2	0	1	0	8
x_7	2	-1	0	-3	0	-1	1	2
z	3	-2	0	2	0	3	0	24

$\longrightarrow$

III	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
x_5	-3/4	1	0	0	1/4	-1/2	0	6
x_6	5/4	0	1	2	1/4	1/2	0	14
x_7	5/4	0	0	-3	1/4	-3/2	1	8
z	3/2	0	0	2	1/2	2	0	36

So the optimal solution is $(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, 6, 14, 0, 0, 0, 8)$, with optimal value 36. □

Q.4) Solution:

(a) (1) For $\min\{x_{Bi}/y_{i2}; y_{i2} > 0\} = x_3/y_{22}$, so the leaving variable for x_2 is x_3.
(2) Since there is only one element y_{24} is positive in $\vec{y_4}$, so the leaving variable for x_4 is x_3.
(3) For $\min\{x_{Bi}/y_{i5}; y_{i5} > 0\} = x_1/y_{35}$, so the leaving variable for x_5 is x_1.
(4) There does not exist positive number in y_6, so there is no leaving variable for it.
(5) For $\min\{x_{Bi}/y_{i7}; y_{i7} > 0\} = x_3/y_{27}$, so the leaving variable for x_7 is x_3.

(b) We know that after a pivot on y_{ij}, the new value

$$z' = z + x_{Bi} \times \frac{-(z_j - c_j)}{y_{ij}}$$

where z denotes the old value. So the result of each pivot of above is (1) $z' - z = x_3 \times \frac{-(-5)}{2} = 15$, increase.
(2) $z' - z = x_3 \times \frac{-4}{3} = -8$, decrease.
(3) $z' - z = x_1 \times \frac{-(-1)}{6} = 0$, the new value is equal to the old one.
(5) $z' - z = x_3 \times \frac{0}{3} = 0$, the new value is equal to the old one.

□

Q.5) Solution: First we compute the maximum value of $-x_2 + 3x_3 - 2x_5$ by simplex mehtod.

I	x_1	x_2	x_3	x_4	x_5	x_6	Solution
hline x_1	1	3	-1	0	2	0	7
x_4	0	-2	4*	1	0	0	12
x_6	0	-4	3	0	8	1	10
z	0	1	-3	0	2	0	0

$\longrightarrow$

II	x_1	x_2	x_3	x_4	x_5	x_6	Solution
hline x_1	1	5/2*	0	1/4	2	0	10
x_3	0	-1/2	1	1/4	0	0	3
x_6	0	-5/2	0	-3/4	8	1	1
z	0	-1/2	0	3/4	2	0	9

$\longrightarrow$

III	x_1	x_2	x_3	x_4	x_5	x_6	Solution
hline x_2	2/5	1	0	1/10	4/5	0	4
x_3	1/5	0	1	3/10	2/5	0	5
x_6	1	0	0	-1/2	10	1	11
z	1/5	0	0	4/5	12/5	0	11

From the last tableau, we know that the optimal solution is $(0, 4, 5, 0, 0, 11)^T$, while the minimum value of $x_2 - 3x_3 + 2x_5$ is -11. □

Q.6) Solution: First we denote

$$A = (\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}) = \begin{pmatrix} 5 & 20 & 1 & 0 \\ 5 & -5 & -1 & 1 \end{pmatrix}$$

(a) Since x_3, x_4 are basic variables, then the basic matrix

$$B = (\vec{a_3}, \vec{a_4}) = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

and

$$B^{-1}A = \begin{pmatrix} 5 & 20 & 1 & 0 \\ 10 & 15 & 0 & 1 \end{pmatrix}$$

with the basic solution as

$$(x_3, x_4)^T = B^{-1}b = \begin{pmatrix} 400 \\ 450 \end{pmatrix}$$

and the reduced cost coefficients:

$$c_B B^{-1}A - (c_1, c_2, c_3, c_4) = (0,0)B^{-1}A - (45, 80, 0, 0) = (-45, -80, 0, 0)$$

So we get the initial tableau as

I	x_1	x_2	x_3	x_4	Solution
x_3	5	20	1	0	400
x_4	10*	15	0	1	450
z	-45	-80	0	0	0

Select x_1 as the entering variable, and so the leaving variable is x_4. Pivot on y_{21}, we have

II	x_1	x_2	x_3	x_4	Solution
x_3	0	25/2*	1	-1/2	175
x_1	1	3/2	0	1/10	45
z	0	-25/2	0	9/2	2025

$\longrightarrow$

III	x_1	x_2	x_3	x_4	Solution
x_2	0	1	2/25	-1/25	14
x_1	1	0	-3/25	4/25	24
z	0	0	1	4	2200

Hence the optimal solution is $(24, 14, 0, 0)$ with optimal value 2200.

(b) For the case using x_1, x_2 as basic variables, the basic matrix is

$$B = (\vec{a_1}, \vec{a_2}) = \begin{pmatrix} 5 & 20 \\ 5 & -5 \end{pmatrix}$$

and

$$B^{-1}A = -\frac{1}{125}\begin{pmatrix} -5 & -20 \\ -5 & 5 \end{pmatrix} A = \begin{pmatrix} 1 & 0 & -3/25 & 4/25 \\ 0 & 1 & 2/25 & -1/25 \end{pmatrix}$$

So the basic solution is

$$(x_1, x_2)^T = B^{-1}b = \begin{pmatrix} 24 \\ 14 \end{pmatrix}$$

and the reduced cost coefficients are:

$$c_B B^{-1}A - (c_1, c_2, c_3, c_4) = (45, 80)B^{-1}A - (45, 80, 0, 0) = (45, 80, 1, 4) - (45, 80, 0, 0) = (0, 0, 1, 4)$$

so this basic solution $(24, 14, 0, 0)$ is optimal, and the optimal value is

$$45 \times 24 + 80 \times 14 = 2200$$

□