MAT3210 Assignment 3 Suggested Solution

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Q.1) Solution: Let $x_4 = -x_2, x_3^+ = \max\{x_3, 0\}$ and $x_3^- = \max\{-x_3, 0\}$, we give the standard form as:

$$\begin{array}{ll} \text{Maximize} & z = x_1 + x_3^+ - x_3^- \\ \text{Subject to} & x_1 + x_4 - x_5 = 1 \\ & x_3^+ - x_3^- + x_4 - x_6 = 1 \\ & x_1 + x_3^+ - x_3^- + x_7 = 3 \\ & x_1, x_3^+, x_3^-, x_4, x_5, x_6, x_7 \geq 0 \end{array}$$

□

Q.2) Solution: Let $x_6 = -x_3, x_4^+ = \max\{x_4, 0\}$ and $x_4^- = \max\{-x_4, 0\}$, we give the standard form as:

$$\begin{array}{ll} \text{Maximize} & z' = -x_1 + 2x_2 - x_4^+ + x_4^- \\ \text{Subject to} & x_1 + x_4^+ - x_4^- + x_7 = 1 \\ & x_6 + x_4^+ - x_4^- + x_8 = 1 \\ & x_2 - x_6 + x_9 = 2 \\ & x_5 = 1 \\ & x_1, x_2, x_4^+, x_4^-, x_5, x_6, x_7, x_8, x_9 \geq 0 \end{array}$$

□

Q.3) Solution:

(a) It is easy to check that $\vec{x_0}$ is a FS. And since the number of non-zero variables in $\vec{x_0}$ is 4, ≥ 2, the rank of

$$A = (\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}) = \begin{pmatrix} 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

(b) First, by $\vec{a_1} - 2\vec{a_3} - \vec{a_4} = 0$, we get the new solution as

$$(1,1,1,1)^T - (1,0,-2,-1)^T = (0,1,3,2)^T$$

which is still a non-basic FS. Continuously, by $\vec{a_2} - 3\vec{a_3} - 2\vec{a_4} = 0$, we have

$$(0,1,3,2)^T - (0,1,-3,-2)^T = (0,0,6,4)^T$$

which is a BFS.

□

Q.4) Solution:

(a) The standard form of this problem is:

$$\begin{array}{ll} \text{Maximize} & z' = -3x_1 + x_2 \\ \text{Subject to} & x_1 + 2x_2 + x_3 = 4 \\ & x_1 - x_2 + x_4 = 0 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

(b) Input $x_1 = 1, x_2 = 1$ into the above standard form, we have $x_3 = 1, x_4 = 0$, so the solution $\vec{x_0} = (1,1,1,0)^T$. It has 3 non-zero variables, while the rank of the matrix

$$A = (\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}) = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

is 2, so $\vec{x_0}$ is non-basic.
By $\vec{a_1} + \vec{a_2} - 3\vec{a_3} = 0$, we get a basic FS as:

$$(1,1,1,0)^T - (1,1,-3,0)^T = (0,0,4,0)^T$$

(c) For this degenerate solution, we regard x_3, x_4 as the basic variables, and since $c_3 = c_4 = 0$, we have that

$$(z_1, z_2, z_3, z_4) - (c_1, c_2, c_3, c_4) = (c_3, c_4)B^{-1}A - (-3,1,0,0) = (0,0,0,0) - (-3,1,0,0) = (3,-1,0,0)$$

so this solution is not optimal. We write the initial tableau as following:

Basic	x_1	x_2	x_3	x_4	Solution
x_3	1	2	1	0	4
x_4	1	-1	0	1	0
z'	3	-1	0	0	0

Here we select x_2 as the entering variable, and x_3 as the leaving variable. We have:

Basic	x_1	x_2	x_3	x_4	Solution
x_2	1/2	1	1/2	0	2
x_4	3/2	0	1/2	1	2
z'	7/2	0	1/2	0	2

the solution $(0,2,0,2)^T$ is an improved BFS(we can also find that it is the optimal solution).

(d) See (b) and (c).

□

Q.5) Solution: First, let

$$A = (\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}) = \begin{pmatrix} 2 & 1 & 2 & 4 \\ 3 & -1 & -2 & 6 \end{pmatrix}$$

(a) It is easy to verify that $\vec{x_0}$ is a FS, and since $\vec{a_1}, \vec{a_2}$ are linear independent, x_0 is basic. the basic matrix is

$$B = (\vec{a_1}, \vec{a_2}) = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$$

and

$$(\vec{y_1}, \vec{y_2}, \vec{y_3}, \vec{y_4}) = B^{-1}A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

(b) Since

$$(z_1, z_2, z_3, z_4) = c_B B^{-1}A = (1,4)B^{-1}A = (1,4,8,2)$$

we know that x_0 is not optimal for $z_4 - c_4 = 2 - 5 < 0$.

(c) Certainly x_4 is the entering variable, and y_{14} is the unique positive element in $\vec{y_4}$, which implies that the leaving variable is x_1.

(d) In the following tableau:

Basic	x_1	x_2	x_3	x_4	Solution
x_1	1	0	0	2	3
x_2	0	1	2	0	4
z	0	0	1	-3	19

pivot on y_{14}, we have

Basic	x_1	x_2	x_3	x_4	Solution
x_4	1/2	0	0	1	3/2
x_2	0	1	2	0	4
z	3/2	0	1	0	47/2

So the optimal solution is $(0,4,0,3/2)^T$, with the optimal value 47/2.

□

Q.6) Solution: Let

$$A = (\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}) = \begin{pmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ 2 & -4 & 0 & 0 & 1 & 0 \\ -4 & -3 & 8 & 0 & 0 & 1 \end{pmatrix}$$

(a) It is easy to see that $\vec{x_0}$ is a FS, and since $\vec{a_4}, \vec{a_5}, \vec{a_6}$ are linear independent, it is also basic. The basic matrix is

$$B = (\vec{a_4}, \vec{a_5}, \vec{a_6}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

and

$$(\vec{y_1}, \vec{y_2}, \vec{y_3}, \vec{y_4}, \vec{y_5}, \vec{y_6}) = B^{-1}A = A$$

(b) Since $c_4 = c_5 = c_6 = 0$, we have

$$(z_1, z_2, z_3, z_4, z_5, z_6) - (c_1, c_2, c_3, c_4, c_5, c_6) = -(c_1, c_2, c_3, c_4, c_5, c_6) = (-3, -4, 0, 0, 0, 0)$$

so $\vec{x_0}$ is not optimal.

(c) Here we select x_1 as the entering variable, and by

$$\min\{x_{Bi}/y_{i1} : y_{i1} > 0\} = \min\{x_4/y_{11}(= 7/3), x_5/y_{21}(= 6)\} = x_4/y_{11}$$

the leaving variable is x_4.

□

Remark. We also can select x_2 as the entering variable, and since there is no positive element in $\vec{y_2}$, so there does not exist leaving variable, that is, there is no optimal value of this problem.