Solution Keys to MAT3210 Solution 10

1.Solution

(a)

$$\min\ 2x_{11} + 3x_{12} + 4x_{13} + x_{14} + x_{21} + 3x_{22} + 5x_{23} + 6x_{24}$$

subject to

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 20 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 30 \\ x_{11} + x_{21} &= 10 \\ x_{12} + x_{22} &= 20 \\ x_{13} + x_{23} &= 10 \\ x_{14} + x_{24} &= 10 \\ x_{ij} \geq 0\ for\ all\ i, j \end{aligned}$$

(b) $$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$rank(A) = n + m - 1 = 2 + 4 - 1 = 5$

(c) The dual problem :

$$\max 20u_1 + 30u_2 + 10v_1 + 20v_2 + 10v_3 + 10v_4$$

subject to

$$u_i,\ \ v_j\ \ free \quad i=1\,,2 \quad j=1\,,\ \cdots\,,\ 4$$

$$u_1+v_1 \leq 2$$

$$u_1+v_2 \leq 3$$

$$u_1+v_3 \leq 4$$

$$u_1+v_4 \leq 1$$

$$u_2+v_1 \leq 1$$

$$u_2+v_2 \leq 3$$

$$u_2+v_3 \leq 5$$

$$u_2+v_4 \leq 6$$

(d)We take in order the $4-th$, $8-th$, $1-th$, $2-th$, $3-th$ columns and $1-th$, $2-th$, $3-th$, $4-th$, $5-th$ rows of A to consist a square matrix

$$B=\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(e) The columns we chosen in (d) are a_{14} , a_{21} , a_{22} , a_{23} , a_{24}

$a_{11}=a_{14}-a_{24}+a_{21}$, $a_{12}=a_{14}-a_{24}+a_{22}$, $a_{13}=a_{14}-a_{24}+a_{23}$

2.Solution

We choose x_{11} , x_{12} , x_{22} , x_{23} , x_{24} as the basic variables

then let the nonbasic variables equal 0 , we can get a BFS (10 , 10 , 0 , 0 , 0 , 10 , 10 , 10)

$$B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$a_{13} = a_{12} - a_{22} + a_{23}$, $a_{14} = a_{12} - a_{22} + a_{24}$, $a_{21} = a_{22} - a_{12} + a_{11}$

$z_{ij} - c_{ij} = \sum_{x_{\alpha\beta} \in B} y^{ij}_{\alpha\beta} c^B_{\alpha\beta} - c_{ij}$

We get $z_{13} - c_{13} = 1$, $z_{14} - c_{14} = 5$, $z_{21} - c_{21} = 1$

We should choose the variable x_{ij} with $z_{ij} - c_{ij} > 0$. Then we can choose any one of x_{13} , x_{14} , x_{21} as the entering variable .

If we choose x_{14} as the entering variable , $x_{B12} = x_{B24} = \min\{x_{B\alpha\beta} | y^{14}_{\alpha\beta} = 1\}$ hence we can choose x_{24} or x_{12} as the leaving variable .

3.Solution

(a) rank(A)=3+4-1=6

(b)

(0)

10	2	20	11	15
12	7	9	20	25
4	14	16	18	10
5	15	15	15	

(1)

1 (top)		1			
	[10]	[2] 15	[20]	[11]	0
	[12]	[7]	[9]	[20]	25
	[4]	[14]	[16]	[18]	10
	5	0	15	15	

(2)

	2	1			
	[10]	[2] 15	[20]	[11]	0
	[12]	[7]	[9]	[20]	25
	[4] 5	[14]	[16]	[18]	5
	0	0	15	15	

(3)

	2	1	3		
	[10]	[2] 15	[20]	[11]	0
	[12]	[7]	[9] 15	[20]	10
	[4] 5	[14]	[16]	[18]	5
	0	0	0	15	

(4)

	2	1	3	4	
	[10]	[2] 15	[20]	[11] 0	0
	[12]	[7]	[9] 15	[20]	10
	[4] 5	[14]	[16]	[18]	5
	0	0	0	15	

(5)

	2	1	3	4	
	[10]	[2] 15	[20]	[11] 0	0
	[12]	[7]	[9] 15	[20]	10
5	[4] 5	[14]	[16]	[18] 5	0
	0	0	0	10	

(6)

	2	1	3	4	
	$\boxed{10}$	$\boxed{2}$ 15	$\boxed{20}$	$\boxed{11}$ 0	0
6	$\boxed{12}$	$\boxed{7}$	$\boxed{9}$ 15	$\boxed{20}$ 10	0
5	$\boxed{4}$ 5	$\boxed{14}$	$\boxed{16}$	$\boxed{18}$ 5	0
	0	0	0	0	

The basic variables we choose are x_{12} , x_{14} , x_{23} , x_{24} , x_{31} , x_{34}

The starting **BFS** is given by $x_{12} = 15$, $x_{14} = 0$, $x_{23} = 15$, $x_{24} = 10$, $x_{31} = 5$, $x_{34} = 5$

(c) The starting objective value is

$\sum_i \sum_j c_{ij}^B x_{ij}^B = 2 \times 15 + 11 \times 0 + 9 \times 15 + 20 \times 10 + 4 \times 5 + 18 \times 5 = 475$

(d)

$\boxed{10}$	$\boxed{2}$ 15	$\boxed{20}$	$\boxed{11}$ 0
$\boxed{12}$	$\boxed{7}$	$\boxed{9}$ 15	$\boxed{20}$ 10
$\boxed{4}$ 5	$\boxed{14}$	$\boxed{16}$	$\boxed{18}$ 5

The basic matrix B is consisted of column vectors corresponding to the basic variables , $B = (\overrightarrow{a}_{12} , \overrightarrow{a}_{14} , \overrightarrow{a}_{23} , \overrightarrow{a}_{24} , \overrightarrow{a}_{31} , \overrightarrow{a}_{34})$

$$B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Use the "loop method " to get other columns of A in terms of columns of B , and compute $z_{ij} - c_{ij}$, $a_{11} = a_{14} - a_{34} + a_{31}$, $z_{11} - c_{11} = c_{14} - c_{34} + c_{31} - c_{11} = -13$

$a_{13} = a_{14} - a_{24} + a_{23}$, $z_{13} - c_{13} = c_{14} - c_{24} + c_{23} - c_{13} = -20$

$a_{21} = a_{24} - a_{34} + a_{31}$, $z_{21} - c_{21} = c_{24} - c_{34} + c_{31} - c_{21} = -6$

$a_{22} = a_{24} - a_{14} + a_{12}$, $z_{22} - c_{22} = c_{24} - c_{14} + c_{12} - c_{22} = 4$

$a_{32} = a_{34} - a_{14} + a_{12}$, $z_{32} - c_{32} = c_{34} - c_{14} + c_{12} - c_{32} = -5$

$a_{33} = a_{34} - a_{24} + a_{23}$, $z_{33} - c_{33} = c_{34} - c_{24} + c_{23} - c_{33} = -9$

(e)From the value of $z_{ij} - c_{ij}$, we should choose x_{22} as the entering variable

$x_{24} = \min\{x_{B\alpha\beta} | y^{22}_{\alpha\beta} = 1\}$, then we need to choose x_{24} as the leaving variable .

$\boxed{10}$	$\boxed{2}$ $15-\theta$	$\boxed{20}$	$\boxed{11}$ $0+\theta$
$\boxed{12}$	$\boxed{7}$ θ	$\boxed{9}$ 15	$\boxed{20}$ $10-\theta$
$\boxed{4}$ 5	$\boxed{14}$	$\boxed{16}$	$\boxed{18}$ 5

We choose $\theta = 10$ then update the transportation table .

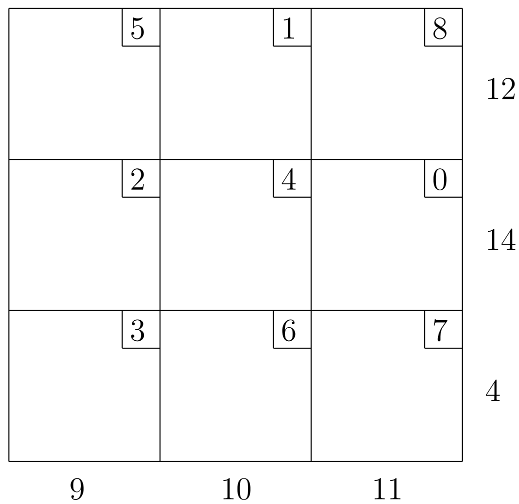
$\boxed{10}$	$\boxed{2}$ 5	$\boxed{20}$	$\boxed{11}$ 10
$\boxed{12}$	$\boxed{7}$ 10	$\boxed{9}$ 15	$\boxed{20}$
$\boxed{4}$ 5	$\boxed{14}$	$\boxed{16}$	$\boxed{18}$ 5

4.Solution

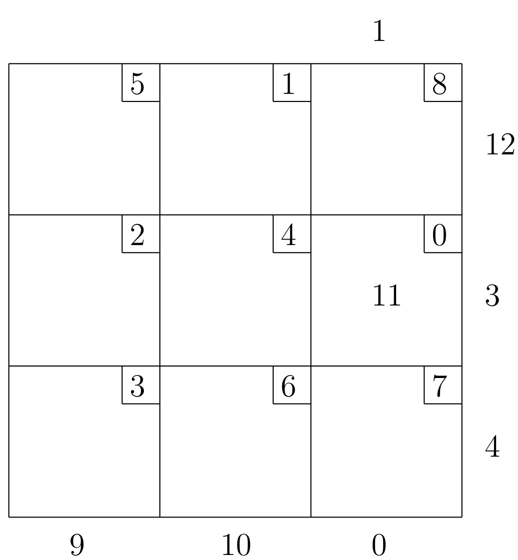
(a) rank(A)=3+3-1=5

(b)

(0)



(1)



(2)

		2	1	
	[5]	[1] 10	[8]	2
	[2]	[4]	[0] 11	3
	[3]	[6]	[7]	4
	9	0	0	

(3)

		2	1	
	[5]	[1] 10	[8]	2
3	[2] 3	[4]	[0] 11	0
	[3]	[6]	[7]	4
	6	0	0	

(4)

		2	1	
	$\boxed{5}$	$\boxed{1}$ 10	$\boxed{8}$	2
3	$\boxed{2}$ 3	$\boxed{4}$	$\boxed{0}$ 11	0
4	$\boxed{3}$ 4	$\boxed{6}$	$\boxed{7}$	0
	2	0	0	

(5)

		2	1	
5	$\boxed{5}$ 2	$\boxed{1}$ 10	$\boxed{8}$	0
3	$\boxed{2}$ 3	$\boxed{4}$	$\boxed{0}$ 11	0
4	$\boxed{3}$ 4	$\boxed{6}$	$\boxed{7}$	0
	0	0	0	

The basic variables we choose are x_{11} , x_{12} , x_{21} , x_{23} , x_{31}

The starting **BFS** is given by $x_{11} = 2$, $x_{12} = 10$, $x_{21} = 3$, $x_{23} = 11$, $x_{31} = 4$

(c) The starting objective value is $\sum_i \sum_j c_{ij} x_{ij} = 38$

(d)

$\boxed{5}$ 2	$\boxed{1}$ 10	$\boxed{8}$
$\boxed{2}$ 3	$\boxed{4}$	$\boxed{0}$ 11
$\boxed{3}$ 4	$\boxed{6}$	$\boxed{7}$

The basic matrix B is consisted of column vectors corresponding to the basic variables , $B = (\vec{a}_{11}\ ,\ \vec{a}_{12}\ ,\ \vec{a}_{21}\ ,\ \vec{a}_{23}\ ,\ \vec{a}_{31})$

$$B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Use the "loop method " to get other columns of A in terms of columns of B , and compute $z_{ij} - c_{ij}$, $a_{13} = a_{11} - a_{21} + a_{23}$, $z_{13} - c_{13} = c_{11} - c_{21} + c_{23} - c_{13} = -5$

$a_{22} = a_{21} - a_{11} + a_{12}$, $z_{22} - c_{22} = c_{21} - c_{11} + c_{12} - c_{22} = -7$

$a_{32} = a_{31} - a_{11} + a_{12}$, $z_{32} - c_{32} = c_{31} - c_{11} + c_{12} - c_{32} = -7$

$a_{33} = a_{31} - a_{21} + a_{23}$, $z_{33} - c_{33} = c_{31} - c_{21} + c_{23} - c_{33} = -6$

(e)From $z_{ij} - c_{ij} \leq 0$, for all ij we claim that the **BFS** is already optimal .