Department of Mathematics 數學系
The Chinese University of Hong Kong 香港中文大學

The Croucher Foundation Advanced Study Institute
Recent Development in Nonlinear Partial Differential Equations: Part I

Date: 3 March 2011
Venue: Rm. 501a, Academic Building 1, IMS, CUHK

Time / Date	3 March 2011 (Thursday)
09:00am – 10:15am	**Professor Manuel del Pino** *University of Chile* Title: New Entire Solutions of Semilinear Elliptic Equations (Part III)
10:15am – 10:45am	Tea Break
10:45am – 12:00pm	**Professor Michael J. Ward** *University of British Columbia* Title: Traps, Patches, Defects, and Spots: An Asymptotic Analysis of Localized Solutions to Some Diffusive and Reaction-Diffusion Systems (Part I)
12:00pm – 02:00pm	Working Lunch#
02:00pm – 02:45pm	**Professor Feng Zhou** *East China Noraml University* Title: Regularity of the extremal solution for some elliptic problems
02:45pm – 03:30pm	**Professor Yaping Wu** *Capital Normal University* Title: The Stability of Travelling Waves with Algebraic Decay for Autocatalytic Reaction Systems
03:30pm – 04:00pm	Tea Break
04:00pm – 04:45pm	**Professor Matthias Winter** *Brunel University* Title: Stability of Cluster Solutions in a Tritrophic Food Chain Model
04:45pm – 05:30pm	**Professor Theodore Kolokolnikov** *Dalhousie University* Title: Instability thresholds for a cross-diffusion model and for a crime model
06:00pm – 08:00pm	Banquet# (Venue: Chung Chi College Staff Club, CUHK)

 # For invited speakers and invited guests only.

New Entire Solutions of Semilinear Elliptic Equations

Professor Manuel del Pino
Departamento de Ingenieria Matemática, Universidad de Chile

Abstract

We will survey some recent results on construction of entire solutions of semilinear elliptic equations. We will mostly focus on the construction of families of solutions to the Allen-Cahn equation of phase transitions, whose level sets suitable scaled concentrate around a given minimal surface. To do so, we shall introduce an infinite-dimensional form of Lyapunov-Schmidt reduction suitable for this and various related questions.

Traps, Patches, Defects, and Spots: An Asymptotic Analysis of Localized Solutions to Some Diffusive and Reaction-Diffusion Systems

Professor Michael J. Ward
Department of Mathematics, University of British Columbia

Abstract

A survey of the development of a unified singular perturbation methodology to analyze some linear and nonlinear PDE models of diffusion and reaction-diffusion type with localized solutions is presented. Specific results from this theory are given for four diverse applications.

The first problem is to determine the mean first passage time (MFPT) for free diffusion from within a sphere to small localized traps on its boundary. In the context of cellular signal transduction, the results predict the time-scale needed for a diffusing molecule to arrive at localized signalling compartments on the boundary of a biological cell. From a mathematical viewpoint, the problem of optimizing this MFPT is shown to be closely related to the well-known problem of finding the minimum energy configuration of repelling point charges on the surface of a sphere.

Secondly, in the context of spatial ecology, a long-standing problem is to determine the persistence threshold for extinction of a species in a heterogeneous spatial landscape consisting of either favorable or unfavorable local habitats. For a 2-D spatial landscape consisting of such localized patches, and in the context of the diffusive logistic model, this extinction threshold is calculated asymptotically and the effects of both habitat fragmentation and habitat location on the persistence threshold are obtained. From a mathematical viewpoint, the persistence threshold represents the principal eigenvalue of an indefinite weight singularly perturbed eigenvalue problem.

Thirdly, the dynamics, stability, and self-replication behavior of localized spot-type solutions to the well-known Gray-Scott reaction-diffusion model of chemical physics in a two-dimensional domain are discussed. Reduced ODE systems for the dynamics of spots are given together with phase diagrams in parameter space classifying the different types of spot instabilities.

Finally, in a bounded 2-D domain, we construct solutions exhibiting point-concentration behavior for a fourth order nonlinear eigenvalue problem for the Biharmonic operator with inverse square law nonlinearity. This problem models the deflection of a micro-plate in due to a voltage bias between the deflectable surface and a rigid ground plate. These nearly singular solutions characterize those portions of the bifurcation diagram for which the nonlinearity is, essentially, localized in the domain.

Regularity of the extremal solution for some elliptic problems

Professor Feng Zhou
Department of Mathematics, East China Normal University

Abstract

In this talk, I first recall some regularity results about the extremal solutions u^* for some semilinear elliptic equation $-\Delta u = \lambda f(u)$ posed on a bounded smooth domain of $\mathbb{R}^n$ with Dirichlet boundary condition. Here f is a C^1 positive nondecreasing convex function on $[0, \infty)$ which is superlinear at infinite. Then I will investigate the regularity of the extremal solution for the equation with advection $-\Delta u + c(x) \cdot \nabla u = \lambda f(u)$ with Dirichlet boundary condition. Now f is a positive nondecreasing convex function, exploding at a finite value $a \in (0, \infty)$. We show that the extremal solution is regular in low dimensional case. In particular, we prove that for the radial case, all extremal solutions are regular in dimension two. This is a joint work with X.Luo and D.Ye.

The Stability of Travelling Waves with Algebraic Decay for Autocatalytic Reaction Systems

Professor Yaping Wu
Department of Mathematics, Capital Normal University

Abstract

It's a joint work with Yi Li, Univ. of Iowa.

Consider the following autocatalytic chemical reaction system

$$\begin{cases} u_t = u_{xx} - u^q v^p, \\ v_t = dv_{xx} + u^q v^p. \end{cases}$$

For $p \geq 1$, $q \geq 1$ and $d > 0$, it is known that there exists a critical speed $c^*(p, q, d)$ such that for any $c \geq c^*(p, q)$ there exist travelling front solutions $(u(x - ct), v(x - ct))$ connecting $(0, 1)$ and $(1, 0)$. For the cases $p > 1$ or $q > 1$, the travelling waves with noncritical speed decay algebraically in space at $+\infty$ or $-\infty$.

In this talk we shall be more interested in the asymptotic stability of the waves with noncritical speed and algebraic spacial decay for $p > 1$ and $q \geq 1$. We shall first talk about our recent work on the asymptotic stability of the waves with algebraic decay in some polynomially weighted spaces for the system with equal diffusion rates. Further we shall talk about our recently obtained abstract results on the existence and analyticity of Evans function for the more general ODE systems with slow algebraic decaying coefficients, and our recent work on the linear and nonlinear exponential stability of waves with algebraic decay when the diffusion rates are close.

Stability of Cluster Solutions in a Tritrophic Food Chain Model

Professor Matthias Winter
Department of Mathematical Sciences, Brunel University

Abstract

We study a tri-trophic food chain model which is based on Schnakenberg type kinetics. It is realistic in a predator-prey context if cooperation of predators is prevalent in the system. In this food chain model there is one predator feeding on the prey and a second predator feeding on the first predator. This means that the first predator plays a hybrid role: it acts both as predator and prey. It is assumed that the prey diffuses much faster than the first predator and that the first predator diffuses much faster than the second predator.

We construct single cluster solutions on an interval for which the profile of the second predator is that of the commonly observed spike in the Schnakenberg model. However, for the first predator a new cluster-type profile is observed which comes from the fact that it acts as predator and prey simultaneously. Its profile in leading order is that of two spikes glued together and connected by a thin transition layer.

We study the stability properties of this solution in terms of the system parameters. We use a rigorous analysis for the linearized operator around single cluster solutions based on nonlocal eigenvalue problems. The following results are established: cluster solutions may be stable or unstable. In particular, it is shown that these patterns are stable if the feed rates and time relaxation constants are both small enough. This is joint work with Juncheng Wei.

Instability thresholds for a cross-diffusion model and for a crime model

Professor Theodore Kolokolnikov
Department of Mathematics and Statistics, Dalhousie University

Abstract

In the first part of this talk, we consider the Shigesada-Kawasaki-Teramoto model of species segregation. Recently, in the limit of high cross diffusion rate of one species, and small diffusion rate of the other, steady states in the shape of an inverted spike were constructed in one dimension [Lou, Ni and Yotsutani, Discrete and cont. dyn. systems 10(1):435-458 2004; Wu and Xu, preprint, 2010]. In this work we consider the stability properties of such spiky states. In particular, we show that K symmetric spikes are stable if the domain length is sufficiently large. When $K = 2$, the instability of a small eigenvalue is triggered first, resulting in a very slow drift of the two spikes, and eventual disappearance of one of them. When $K > 2$, the primary instability is due to a large eigenvalue, resulting in a quick annhiliation of one of the spikes. We also extend the construction of one dimensional steady states to two dimensions.

In the second part of this talk, we study a similar phenomenon for a model of hot spots in crime developed recently by Bertozzi and collaborators at UCLA. In a certain limit, the crime hot-spots are shown to correspond to a spike-type solution. The stability analysis of this model has some similarities to that of the SKT model, but also big differences.

Joint work with Michael Ward and Juncheng Wei.