THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010 UNIVERSITY MATHEMATICS 2024-2025 Term 1
Suggested Solutions of WeBWork Coursework 2

If you find any typos or errors, please send an email to math1010@math.cuhk.edu.hk

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1. (1 point) Find the domain of the function

$$f(x) = \frac{\sqrt{7+5x}}{x^2-81}$$

and write your answer in the **interval notation**.
Domain: ______
Help: Click here for help entering intervals.

Solution: By the definition of $f(x)$, we have $7+5x \geq 0$ and $x^2-81 \neq 0$. By solving these two inequalities, we get the domain of $f(x)$:$[-1.4, 9) \cup (9, \infty)$.

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2. (1 point) The domain of the function $g(x) = \log_a(x^2-49)$ is
$(-\infty,$ _____ $)$ and $($ _____, $\infty)$.

Solution: By the definition of logarithm, we have $x^2-49>0$. Therefore, the domain is $(-\infty, -7) \cup (7, \infty)$.

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3. (1 point) Given that $f(x) = \frac{1}{x}$ and $g(x) = 4x+5$, calculate
(a) $(f \circ g)(x)$= ______, its domain is all real numbers except ______

(b) $(g \circ f)(x)$= ______ , its domain is all real numbers except ______

(c) $(f \circ f)(x)$= ______, its domain is all real numbers except ______

(d) $(g \circ g)(x)$= ______, its domain is (______, ______)
Note: If needed enter ∞ as *inf* and $-\infty$ as *-inf* .

Solution:(a) $(f \circ g)(x) = \frac{1}{g(x)} = \frac{1}{4x+5}$. So we have $4x+5 \neq 0$, i.e. $x \neq -1.25$.
(b) $(g \circ f)(x) = 4f(x)+5 = \frac{4}{x}+5$. So $x \neq 0$.
(c) $(f \circ f)(x) = \frac{1}{f(x)} = x$. Since $f(x) = \frac{1}{x}$, we have $x \neq 0$.
(d) $(g \circ g)(x) = 4g(x)+5 = 16x+25$. The domain of this function is $(-\infty, \infty)$.

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4. (1 point) Given the functions, $f(x) = \dfrac{x-5}{x-3}$ and $g(x) = \sqrt{x+5}$, find the following domains. Use interval notation. help (intervals)

Domain of f	______
Domain of g	______
Domain of $f+g$	______
Domain of $\dfrac{f}{g}$	______
Domain of $\dfrac{g}{f}$	______
Domain of $f(g(x))$	______
Domain of $g(f(x))$	______

Solution:(1) By the definition of $f(x)$, we have $x-3 \neq 0$. So the domain is $(-\infty, 3) \cup (3, \infty)$.
(2) By the definition of $g(x)$, we have $x+5 \geq 0$. So the domain is $[-5, \infty)$.
(3) $f+g = \frac{x-5}{x-3} + \sqrt{x+5}$. The domain is $[-5, 3) \cup (3, \infty)$.
(4) Since $\frac{f}{g} = \frac{x-5}{(x-3)\sqrt{x+5}}$, we have $x-3 \neq 0$ and $x+5>0$. Hence the domain is $(-5, 3) \cup (3, \infty)$.
(5) Since $\frac{g}{f} = \frac{(x-3)\sqrt{x+5}}{x-5}$, we have $x-3 \neq 0$, $x-5 \neq 0$ and $x+5 \geq 0$. Hence the domain is $[-5, 3) \cup (3, 5) \cup (5, \infty)$.
(6) Since $f(g(x)) = \frac{g(x)-5}{g(x)-3} = \frac{\sqrt{x+5}-5}{\sqrt{x+5}-3}$, we have $x+5 \geq 0$ and $\sqrt{x+5}-3 \neq 0$. Hence the domain is $[-5, 4) \cup (4, \infty)$.
(7) Since $g(f(x)) = \sqrt{f(x)+5} = \sqrt{\frac{x-5}{x-3}+5}$, we have $x-3 \neq 0$ and $\frac{x-5}{x-3}+5 \geq 0$. By solving these inequalities, we obtain the domain of this function:$(-\infty, 3) \cup (\frac{10}{3}, \infty)$.

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5. (1 point) Suppose $f(x) = 8x-7$ and $g(y) = \dfrac{y}{8} + \dfrac{7}{8}$.

(a) Find the composition $g(f(x)) =$ ______ help (formulas)

(b) Find the composition $f(g(y)) =$ ______ help (formulas)

(c) Are the functions f and g inverse to each other? [?/Yes/No]

Solution:(a) $g(f(x)) = \frac{f(x)}{8} + \frac{7}{8} = \frac{8x-7}{8} + \frac{7}{8} = x$.
(b) $f(g(y)) = 8g(y)-7 = 8*(\frac{y}{8}+\frac{7}{8})-7 = y$.
(c) Since $g(f(x)) = x$ and $f(g(y)) = y$, f and g inverse to each other.

6. (1 point) Find the inverse function to $y = f(x) = \dfrac{4-5x}{3-8x}$.

$x = g(y) =$ ________ help (formulas)

Solution:We need to represent x by y by solving this equation: $y = \frac{4-5x}{3-8x}$. We multiple $3-8x$ on the both sides of the equation and simplify: $x(8y-5) = 3y-4$. So we have $x = g(y) = \frac{3y-4}{8y-5}$.

7. (1 point)

Part 1: Evaluate the limit

Evaluate the following limit by simplifying the expression (first answer box) and then evaluating the limit (second answer box).

$$\lim_{x\to 4} \frac{x^2+3x-28}{x-4} = \lim_{x\to 4} \_\_\_ = \_\_\_.$$

Note: In your written solution, you should write the limit statement $\lim\limits_{x\to 4}$ in every step except the last one, where the limit is finally evaluated.

Part 2: Follow-up question

Solution: $\lim_{x\to 4} \frac{x^2+3x-28}{x-4} = \lim_{x\to 4} \frac{(x+7)(x-4)}{x-4} = \lim_{x\to 4}(x+7) = 11$.

8. (1 point) Evaluate the limit

$$\lim_{y\to 6} \frac{\frac{1}{y}-\frac{1}{6}}{y-6}.$$

(If the limit does not exist, enter "DNE".)

Limit = ________

Solution: $\lim_{x\to 6} \frac{\frac{1}{y}-\frac{1}{6}}{y-6} = \lim_{x\to 6} \frac{\frac{6-y}{6y}}{y-6} = \lim_{x\to 6} -\frac{1}{6y} = -\frac{1}{36}$.

9. (1 point)

Let a be a positive real number. Evaluate the limit:

$$\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{8(x-a)} = \_\_\_$$

Solution: $\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{8(x-a)} = \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{8(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})} = \lim_{x\to a} \frac{1}{8(\sqrt{x}+\sqrt{a})} = \frac{1}{16\sqrt{a}}$.

10. (1 point) Determine whether the sequences are increasing, decreasing, or not monotonic. If increasing, enter I as your answer. If decreasing, enter D as your answer. If not monotonic, enter N as your answer.

___1. $a_n = \frac{n-2}{n+2}$
___2. $a_n = \frac{\cos n}{2^n}$
___3. $a_n = \frac{\sqrt{n+2}}{9n+2}$
___4. $a_n = \frac{1}{2n+9}$

Solution:(1) $a_n = \frac{n-2}{n+2} = 1 - \frac{4}{n+2}$. Hence the sequence is increasing.

(2) $a_1 = \frac{cos(1)}{2} > 0$, $a_2 = \frac{cos(2)}{4} < 0$ (since $\frac{\pi}{2} < 2 < \pi$), and $a_7 = \frac{cos(7)}{128} > 0$ (since $2\pi < 7 < \frac{5\pi}{2}$). So the sequence is not monotonic.

(3) Let $b_n = \frac{n+2}{(9n+2)^2}$. Then $a_n = \sqrt{b_n}$. Since $b_{n+1} - b_n = \frac{n+3}{(9n+11)^2} - \frac{n+2}{(9n+2)^2} = \frac{(9n+2)^2(n+3)-(9n+11)^2(n+2)}{(9n+2)^2(9n+11)^2} = -\frac{81n^2+405n+230}{(9n+2)^2(9n+11)^2} < 0$, and the function $y = \sqrt{x}$ is increasing. The sequence is decreasing.

(4) $a_{n+1} - a_n = \frac{1}{2n+11} - \frac{1}{2n+9} = -\frac{2}{(2n+11)(2n+9)} < 0$. Hence the sequence is decreasing.

11. (1 point) Determine whether the sequence $a_n = \dfrac{1^1}{n^2} + \dfrac{2^1}{n^2} + \cdots + \dfrac{n^1}{n^2}$ converges or diverges. If it converges, find the limit.

Note,

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

Converges (y/n): ________

Limit (if it exists, "DNE" otherwise): ________________

Solution: $a_n = \frac{1}{n^2}(\sum_{i=1}^{n} i) = \frac{n(n+1)}{2n^2} = \frac{n+1}{2n}$. So the sequence converges. And $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{2n} = 0.5$.