THE CHINESE UNIVERSITY OF HONG KONG
MATH 1540 Homework Set 4
Due time 6:30 pm Nov 14, 2016

1. Show that:

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x+y-3} = 0.$

   (b) $\lim_{(x,y)\to(1,1)} \dfrac{x^2-y^2}{x-y} = 2.$

   (c) $\lim_{(x,y)\to(1,-1)} \dfrac{x^2-xy-2y^2}{x+y} = 3.$

   (d) $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{xy}$ does not exist.

   (e) $\lim_{(x,y)\to(0,0)} \dfrac{y}{x^3+y}$ does not exist.

   (f) $\lim_{(x,y)\to(0,0)} \dfrac{x^6}{x^4+y^2} = 0.$

   (Hint: Consider using Sandwich Theorem.)

2. (a) Let $f(x,y) = 5x^7 - 2xy^3 + 6$. Show that $f_x(-1,1) = 33$, $f_y(2,2) = -48$.

   (b) Let $f(x,y) = \sqrt{xy - y^2}$. Show that:

$$\left.\frac{\partial f}{\partial x}\right|_{(x,y)=(3,2)} = \frac{2}{2\sqrt{2}}.$$

$$\left.\frac{\partial f}{\partial y}\right|_{(x,y)=(3,2)} = \frac{-1}{2\sqrt{2}}.$$

   (c) Let $f(x,y) = \log_x y, \quad y>0, x\neq 1$. Show that:

$$\left.\frac{\partial f}{\partial x}\right|_{(x,y)=(e,2)} = \frac{-\ln 2}{e}.$$

$$\left.\frac{\partial f}{\partial y}\right|_{(x,y)=(e,2)} = \frac{1}{2}.$$

   (d) Let $f(x,y) = x^y + y^x, \quad x,y>0$. Show that:

$$\left.\frac{\partial f}{\partial x}\right|_{(x,y)=(1,e^2)} = e^2 + 2e^2.$$

3. (a) Let $f(x,y,z) = xz + y^2z + \cos(z)$.
   Via explicit computation of second order partial derivatives, show that:

$$\frac{\partial^2 f}{\partial y \partial z} = \frac{\partial^2 f}{\partial z \partial y}$$

(b) Let $f(x, y, z) = xyz + \sqrt{xz}$.
Via explicit computation of third order partial derivatives, show that:

$$f_{xzy} = f_{yzx}$$

4. Let:

$$f(x, y) = \begin{cases} x^2 y & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Show that:

(a) $f_x(-1, 2) = 0$.
(b) $f_x(3, -7) = -42$.
(c) $f_x(0, y) = 0$, for all $y \in \mathbb{R}$.
(d) $f_{xy}(0, 0) = 0$.

5. (Optional) Let g be a continuous function defined on $\mathbb{R}$. Let $f(x, y) = \displaystyle\int_{xy}^{y} g(t)\, dt$. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.