Deng Yongzhe Suppose $f(x)$ is twice differentiable in $[0, 1]$ and $|f''(x)| \leq M$, and $f(x)$ get its maxima in $(0, 1)$. Try to show: $|f'(0)| + |f'(1)| \leq M$.

Lam Chi Yeung Let $f : \mathbb{R} \to \mathbb{R}$ be a function which satisfies

$$|f(x) - f(y)| \leq (x - y)^2$$

for all $x, y \in \mathbb{R}$. Show that $f$ is a constant.

Ng Tsz Ching Integrate $\int_{-\infty}^{\infty} e^{-x^2} dx$.

Chen Yu What is $1 - 1 + 1 - 1 + 1 - 1 + \cdots = ?$

Chen Tengrove the following properties of traces.

1. $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$;
2. $\text{tr}(kA) = k \text{tr}(A)$;
3. $\text{tr}(A^T) = \text{tr}(A)$;
4. $\text{tr}(AB) = \text{tr}(BA)$.

Liu Beibei Function $f$ satisfies functional equation $f(x + y) = f(x) + f(y)$ ($\forall x, y \in \mathbb{R}$), and $f$ is continuous at $x = 0$, then there is only one solution $f(x) = ax$ satisfying the equation ($a$ is a constant).

Li Hangfan Here are two problems:

1. Integrate $\int \frac{\ln x}{x^5} dx$.
2. Integrate $\int \frac{2 + \sqrt{x}}{3 - \sqrt{x}} dx$.

Xu Ang Show that: $\lim_{n \to \infty} \sqrt[n]{n} = 1$
Choi Ki Kit  Answer the following questions:

1. Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is not analytic at 0.

2. Evaluate $\int \int_{\mathbb{R}^2} e^{-(x^2+y^2)} dx dy$.

3. Let $A$ be a $n \times n$ self-adjoint matrix. Suppose $R(x) = \frac{<Ax,x>}{||x||^2}$, then $\max_{x \neq 0} R(x)$ is the largest eigenvalue of $A$.

Chan Ho Yuan  Answer the following questions:

1. Let

\[
A = \begin{bmatrix}
1 & -1 & -2 & 0 \\
-1 & 1 & 3 & 1 \\
-2 & 2 & 7 & 3 \\
2 & -2 & -6 & -2
\end{bmatrix}
\]

(a) Find $u_1, u_2 \in \mathbb{R}^4$ such that $\text{span}\{u_1, u_2\} = \text{N}(A)$, where $\text{N}(A)$ is the null space of $A$.

(b) Find a $u_3 \in \mathbb{R}^4$ such that $Au_3 = b$, where $b = (3, -4, -9, 8)$.

(c) Find a $u_4 \in \mathbb{R}^4$ such that $Au_4 = b$, where $b = (-2, 3, 7, -6)$.

(d) Show that every $x \in \mathbb{R}^4$ can be written uniquely as a linear combination of $u_1, u_2, u_3, u_4$.

2. (A First Course in Linear Algebra by Robert A. Beezer, P.56, T40)

Suppose $Ax = b$ is a consistent system of linear equations in which two columns of $A$ are equal. Prove that the system has infinitely many solutions.

Zuo Cheng  Show that Let $M$ be a subspace of the Hilbert space $H$. Let $v \in H$

$M$ and define $\delta := \inf \{\|v - w\| : w \in M\}$. (Note that $\delta > 0$ since $M$ is closed in $H$) Then there exists $w_0 \in M$ such that:

(i) $\|v - w_0\| = \delta$, i.e., there exists a closest point $w_0 \in Mtov$, and

(ii) $v - w_0 \in M^\perp$. 

Cheng Siu Hong  This is to show \( \int_0^\infty \frac{\sin(x)}{x} \, dx = \frac{\pi}{2} \) by computing a double integral (using Fubini’s Theorem) and elementary calculus techniques such as integration by parts.

Define the improper integral of an improperly integrable function \( f(x) \) by

\[
\int_a^\infty f(x) \, dx = \lim_{b \to \infty} \int_a^b f(x) \, dx
\]

(a) Show that \( \int_0^\infty \exp(-xy) \sin(x) \, dy = \frac{\sin(x)}{x} \).

(b) Evaluate \( \int_0^a \exp(-xy) \sin(x) \, dy \).

(c) By using Fubini’s theorem and the result of (a) and (b), show that \( \int_0^\infty \frac{\sin(x)}{x} \, dx = \frac{\pi}{2} \).

Wang Chuiji  Show that \( A,B \) are commutative matrices, and they both can be diagonalized, then they can be diagonalized simultaneously.

Choi Chi Po  For \( m = 1, 2, 3, \ldots \), \( n = 1, 2, 3, \ldots \), let

\[ s_{m,n} = \frac{m}{m + n} \]

Compute

\[ \lim_{n \to \infty} \lim_{m \to \infty} s_{m,n} \]

and

\[ \lim_{m \to \infty} \lim_{n \to \infty} s_{m,n} \]

Do they have the same value? Explain your answer.

Wen Jia  Find the first five derivatives of the following functions:

1. \( f(x) = \frac{1}{2 - x} \)
2. \( f(x) = \ln(3 + x) \)

Yin Guojian  Answer the following questions:

1. Using L’Hospital’s Rule to evaluate \( \lim_{x \to 0} \frac{(1 - \cos x) \sin 4x}{x^3 \cos x} \).
2. Find \( \int e^{2x} \sin x \, dx \).
Luo Tianwen Compute the following limit

$$\lim_{{x \to 0^+}} x \ln x$$

Liu Xin Prove that: in an n-dimensional real Euclidean space, the operator $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ does not change under rotation.

Du Yangge Find the following limit by Riemann integral:

$$\lim_{{n \to \infty}} \sum_{k=0}^{n-1} \frac{k}{n^2} \sin\left(\frac{k}{n}\right).$$

Kong Shilei Let $a$, $b$, $c$, $d$ be some real numbers such that the limit

$$\lim_{{x \to 0}} \frac{\sin^2 2x + a + bx + cx^2 + dx^3}{x^4}$$

exists. Find the values of $a$, $b$, $c$, $d$ and the limit.

Mei Yu Find $\lim_{{x \to 0}} x^\sin x$.

Lee Man Chun Show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(Hint: consider $\int_{0}^{1} \int_{0}^{1} \frac{1}{1-xy} dxdy$).

Min Jie Let $A$ be an $n \times n$ matrix and $A \ast A = A \ast A^t$. Show that $A$ is symmetric. (Hint: use induction on the dimension of $A$).

Tao Ran Show that if $x \in \mathbb{R}$, $y \in \mathbb{R}$, and $x < y$, then there exists a rational number $p \in \mathbb{Q}$ such that $x < p < y$.

Yuan Zhiri As we can see, for continuous functions, the existence of

$$\int_{0}^{\infty} f(x)dx \text{ and } \lim_{{x \to \infty}} f \text{ are somehow related. If } \lim_{{x \to \infty}} f \text{ does not equal to zero, then } \int_{0}^{\infty} f(x)dx \text{ makes no sense. So, will } \lim_{{x \to \infty}} f \text{ equal to zero if } \int_{0}^{\infty} f(x)dx < \infty?$$
Yuan Yuan Considering \( s_n = \sum_{k=1}^{n} \frac{1}{k!} \), it is easy to prove \( s_n < 2 \). So we can define
\[
\lim_{n \to \infty} s_n = e. \tag{1}
\]
Prove
\[
\lim_{n \to \infty} (1 + \frac{1}{n})^n = e. \tag{2}
\]

Zhang Pengfei Show that if \( A, B \) are two \( n \times n \) matrices, then
\[
\det \begin{bmatrix} A & B \\ B & A \end{bmatrix} = \det(A + B) \cdot \det(A - B).
\]

Xiao Yao Calculate the integral of \( \int_{-\infty}^{\infty} e^{-x^2} dx \)

Chen Guanheng A certain ecological territory contains \( S \) thousands squirrels and \( R \) thousands rabbits. Currently, there are 4000 of each species, and the grow rates of the population with respect to time satisfies the following equations:
\[
\begin{align*}
\frac{dR}{dt} &= 63R - 3RS, \\
\frac{dS}{dt} &= 26S - RS.
\end{align*}
\]
Find the relationship of \( R \) and \( S \).

Liu Haixia Function \( f \) satisfies functional equation \( f(x + y) = f(x) + f(y) \) (\( \forall x, y \in \mathbb{R} \)), and \( f \) is continuous at \( x = 0 \), then there is only one solution \( f(x) = ax \) satisfying the equation (\( a \) is a constant).
Liu Keji

1. Determine the domain of the given function

\[ f(t) = \frac{t + 2}{\sqrt{9 - t^2}} \]

2. Find the composite function \( f(g(x)) \).

(1) \( f(u) = 3u^2 + 2u - 6, \quad g(x) = x + 2 \),
(2) \( f(u) = (u - 1)^3 + 2u^2, \quad g(x) = x + 1 \).

3. Find functions \( h(x) \) and \( g(u) \) such that \( f(x) = g(h(x)) \).

(1) \( f(x) = (x - 1)^2 + 2(x - 1) + 3 \),
(2) \( f(x) = \frac{1}{x^2 + 1} \).

4. Write an equation for the line with the given properties.

(1) Through (5,-2) with slope \(-\frac{1}{2}\).
(2) Through (1,5) and (3,5).
(3) Through (3,5) and perpendicular to the line \( x + y = 4 \).

5. Find the indicated limit if it exists.

(1) \( \lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5} \).
(2) \( \lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2} \).
(3) \( \lim_{x \to +\infty} \frac{x^2 - 2x + 3}{2x^2 + 5x + 1} \).
Ruan Pengfei Prove

\[
\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{\frac{1}{n}x} = 1.
\]

Fangqiong JIAN Compute \( \int \sec x \, dx \).

Dai Lipeng Compute \( \lim_{x \to 0} \frac{x^2y^2}{x^3+y^3} \) as \( x \to 0, y \to 0 \).

Zhao Rui A cyclic curve \( L \) is given by polar coordinate \( r = 1 + \cos \theta, 0 \leq \theta \leq \frac{\pi}{2} \) and segment \([0, 2]\) on \( x \) axis and segment \([0, 1]\) on \( y \) axis. Find the volume of the solid by rotating \( L \) around the \( x \) axis.