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Sums, point counts and polynomial methods

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Abstract

Understanding oscillatory integrals are crucial to many problems in analysis and geometry. There are discrete analogues of oscillatory integrals known as exponential sums and character sums, which often appears in problems in number theory, such as representation by quadratic forms, spectral theory of automorphic forms and automorphic L-functions. In both cases, the goal is to capture as much cancellation as possible. In this talk, we will discuss the particular instances when the sums are over finite fields. We will see that why estimating these sums intimately relates to point count of varieties over finite fields. There was tremendous progress in point counting thanks to the advancement of algebraic geometry in the 20th century. This program was first proposed by Weil and subsequently developed by Dwork, Grothendieck and Deligne. However, the use of algebraic geometry will not be the main focus of this talk. Instead, we will discuss the more elementary Bombieri-Stepanov method, which is based on clever, strategic constructions of auxiliary polynomials of small enough degrees. Stepanov settled the problem for hyperelliptic curves elementarily in which he was able to recover the well-known Hasse-Weil bound. Bombieri extended his method to all algebraic curves (still fall short of the result by algebraic geometry). We will stick to the case of hyperelliptic curves in which things are completely explicit. From this, we deduce the celebrated Weil bound for Kloosterman sums as an example. If time allows, we will also talk about how to to study incomplete sums in which their estimations do not follow very directly from point counting. Everyone is welcome to join!

Date: 5 December 2017 (Tuesday)
Time: 2:30pm – 3:30pm
Venue: Room C2, Lady Shaw Building, The Chinese University of Hong Kong, Shatin

All are Welcome