THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2017 Spring MATH2230
Homework Set 2 (Due on Jan. 29)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P30

1. Find the square roots of (a) $2i$; (b) $1-\sqrt{3}i$ and express them in rectangular coordinates.

2. Find the three cube roots c_k $(k = 0, 1, 2)$ of $-8i$. Express them in rectangular coordinates and plot them in rectangular plane.

P44

8. Sketch the region onto which the sector $r \le 1$, $0 \le \theta \le \pi/4$ is mapped by the transformation (a) $w = z^2$; (b) $w = z^3$; (c) $w = z^4$.

P54

1. Use definition (2). Sec. 15. of limit to prove that
(a) $\lim_{z \to z_0} Re\, z = Re\, z_0$; (b) $\lim_{z \to z_0} \overline{z} = \overline{z_0}$; (c) $\lim_{z \to 0} \frac{\overline{z}^2}{z} = 0$.

5. Show that the function

$$f(z) = \left(\frac{z}{\overline{z}}\right)^2$$

has the value 1 at all nonzero points on the real and imaginary axes, where $z = (x, 0)$ and $z = (0, y)$ respectively, but that it has the value -1 at all nonzero points on the line $y = x$ where $z = (x, x)$. Thus show that the limit of $f(z)$ as z tends to 0 does not exist.

P61-62

8. Use the method in Example 2. Sec. 19. to show that $f'(z)$ does not exist at any point z when (a) $f(z) = Re\, z$; (b) $f(z) = Im\, z$.

9. Let f denote the function whose values are

$$f(z) = \begin{cases} \overline{z}^2/z & \text{when } z \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Show that if $z = 0$, then $\Delta w/\Delta z = 1$ at each nonzero point on the real and imaginary axes in the Δz or $\Delta x \Delta y$ plane. Then show that $\Delta w/\Delta z = -1$ at each nonzero point$(\Delta x, \Delta x)$ on the line $\Delta y = \Delta x$ in that plane. Conclude from these observations that $f'(0)$ does not exist. Note that to obtain this result, it is not sufficient to consider only horizontal and vertical approaches to the origin in the Δz plane.