THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2060B Mathematical Analysis II (Spring 2018)
HW7 Solution

1. (P.223 Q2) Let $E = \{a, b\}$, it is straight forward to check H_n is continuous on $[a, b]$, $H_n'(x) = x^n$ for $x \in [a, b]\backslash E$ and $x^n \in C[a, b] \subset \mathcal{R}[a, b]$. Fundamental theorem of calculus 7.3.1 is thus applicable and the result follows.

2. (P.223 Q3) Separate $[-2, 3]$ into 3 regions $I = [-2, -1]$, $II = [-1, 1]$, $III = [1, 3]$.
   For $|x| \geq 1$, $G(x) = \frac{1}{2}(x^2 - 1)$, further, for $|x| > 1$, $G'(x) = x = g(x)$.
   For $|x| < 1$, $G(x) = \frac{1}{2}(1 - x^2)$, $G'(x) = -x = g(x)$.
   For $x \in [-2, 3]$, $g(x)$ only has finite discontinuity (at 1 and -1), hence $g(x) \in \mathcal{R}[-2, 3]$.
   Let $E_I = \{-1\}$, $E_{II} = \{-1, 1\}$ and $E_{III} = \{1\}$, fundamental theorem of calculus 7.3.1 is thus respectively applicable on I, II, III. and therefore
   $$\int_{-2}^{3} g = \int_{-2}^{-1} g + \int_{-1}^{1} g + \int_{1}^{3} g = G(-1) - G(-2) + G(1) - G(-1) + G(3) - G(1) = G(3) - G(-2) = \frac{5}{2}.$$