Home Work and Quiz II, Friday, April 15

Problem 1
(1) Show that $|x|_X^2$ is convex where X is a normed space.
(2) $\partial\frac{1}{2}|x|^2 = F(x)$, where F is the duality map.
(3) Let $X = W_0^{1,p}(\Omega)$ with norm $|u|_X = (\int_\Omega |\nabla u|^p\,dx)^{1/p}$. Find $F(x)$
(4) Consider the minimization

$$\min \quad \int_\Omega (a(x)\,|\nabla u(x)|^p\,dx - f(x)u(x))\,dx$$

over $u \in W_0^{1,p}(\Omega)$. Derive the necessary optimality.
(5) Consider

$$\frac{\partial u}{\partial t} \in \partial\varphi(u)$$

where φ is a convex and coercive functional on $H_0^1(\Omega)$. Apply the Crandall-Liggett theory on $X = L^2(\Omega)$.

Problem 2 Apply the Grandall-Liggett theory with $x + L^2(\Omega)$ for the heat equation

$$\frac{\partial u}{\partial t} = \Delta u, \quad -\frac{\partial u}{\partial n} \in \beta(u) \text{ at } \partial\Omega$$

where $b : R \to R$ is maximum monotone.