Quiz I, Friday 19, 9:30-12:AM

— Come to the class and discuss the problems

— write your own report

Problem 1 Let $X = C[0,1]$ be the space of continuous functions with sup norm. Then show that $X^* = BV(0,1)$ = the space of (right continuous) bounded variation functions on $[0,1]$, i.e. for every $f \in X^*$ there exists $\nu \in BV(0,1)$ such that $f(x) = \int_0^1 x(t)\,d\nu(t)$ (Riemann Stieltjes integral) for all $x \in X$. $\delta_{t_0} \in X^*$ (i.e. $\delta_{x_0}(\phi) = \phi(t_0)$ for $\phi \in X$). and $\delta_{t_0} \in F(x)$ for $t_0 \in [0,1]$ satisfying $x(t_0) = \max_{t\in[0,1]} |x(t)|$.

Problem 2 Let A be a closed linear operator on a Banach space. $D(A) = dom\,(A)$ is a Banach space with the graph norm

$$|x|_{D(A)} = |x|_X + |Ax|_X.$$

Problem Let $c \in L^\infty(0,1)$. Define the linear operators $A_1 u = -(c(x)u)_x$ in $X = L^1(0,1)$. and $A_2 u = c(x)u_x$ in $X = L^p(0,1)$.

(a) Find $dom(A_2)$ so that A_2 is ω-dissipative. — Hint: $c' \leq M$ (bounded above) if $p > \infty$. If $p = \infty$, then no condition is necessary. Inflow $c(0) > 0$ and Outflow $c(0) \leq 0$.

Find $dom(A_1)$ so that A_1 is ω-dissipative. — Hint Assume $c > 0$ (WLOG). Since $cu \in C[0,1]$ one can decompose $[0,1]$ the sub intervals (t_i, t_{i+1}) on which $cu > 0$ or $cu < 0$ and $cu(t_i) = 0$ and let $u^* = sign_0(cu) = sign_0(u)$. Thus, we have

$$(A_1 u, u^*) = \int_0^1 (-(cu)_x u^*(x))\,dx = c(0)|u(0)| - c(1)|u(1)|.$$

(c) In general show that $dom(A_1)$ and $dom(A_2)$ are different (Hint: piecewise constant).

Remark Also, please study the Appendix.