MATH 3060 Mathematical Analysis III

Tutorial 1 (September 14)

The following were discussed in the tutorial this week:

1. Definition of Fourier series and the notion $\sim$.

2. Uniform and pointwise convergence of Fourier series. Tools to determine such convergence, for example, Weierstrass M-test and Dirichlet’s test.

3. Sketch of proof of the following facts:

   **Theorem.** Let $f$ be a $2\pi$-periodic Riemann integrable function on $[-\pi, \pi]$. If the partial sum $S_N(f)$ converges uniformly to $g$ on $[-\pi, \pi]$, then $g$ is continuous on $[-\pi, \pi]$ and $g = f$ at all continuity point of $f$.

   **Corollary.** Let $f$ be a $2\pi$-periodic continuous function on $[-\pi, \pi]$. If $S_N(f)$ converges uniformly on $[-\pi, \pi]$, then it converges uniformly to $f$.

4. We computed the Fourier series of $f(x) := |x|$ on $[-\pi, \pi]$ and showed that it converges uniformly on $[-\pi, \pi]$.

5. Let $-\pi < a < b < \pi$. We computed the Fourier series of $f(x) := \begin{cases} 1 & \text{if } x \in [a, b] \\ 0 & \text{if } x \in [-\pi, a) \cup (b, \pi] \end{cases}$ on $[-\pi, \pi]$ and showed that it converges pointwisely on $[-\pi, \pi]$ but does not converge uniformly on $[-\pi, \pi]$. 