Some Supplemented Problems

1. Let \((\mathbb{Q} \cap A) \subseteq \mathbb{R}\). Then it is finite if and only if it is bounded above.

   \[ \text{Hint for } \leq: \text{ Assume } x \leq a \text{ then } x + n \leq a + (n+1). \]

   Suppose \(A\) is bounded above. Then \(\forall x \leq \sup A\) (by what?) and so \(\forall x \in A\) for some \(n \in \mathbb{N}\).

   Hence

   \[ n + 3 \leq x < n + 0.1 \]

   + Why?

2. If \((\mathbb{Q} \cap A) \subseteq \mathbb{R}\) and is bounded below by \(x \leq 1\), then \(\exists n \in \mathbb{N}\) such that \(\frac{3}{n} < x < \frac{1}{n} + 0.1\).

3. If \((0 + A) \in \mathbb{Z}\) and is bounded below by \(x < 1\), then \(\frac{3}{n} < x < \frac{1}{n} + 0.1\).

   \(A + n : = \{ a + n : a \in A \}\)

   \(\mathbb{N}\) contained in \(\mathbb{N}\), you should also convince yourself (prove it) that if \(A \subseteq \mathbb{N}\) and \(0 < a \in A\) then \(A \subseteq \mathbb{N}\).
4. If $(\emptyset \neq) A \subseteq N$ (A may be infinite)
   then $A$ has the smallest element.

   Hint: (If finite then use M.I.)
   Pick $a' \in A$ and let
   $A_0 = \{ a \in A : a \leq a' \}$

   Then $A_0$ has the smallest element,
   to be denoted by $a_0$. Check $a_0$ is
   also the smallest element of $A$.

5. If $(\emptyset \neq) A \subseteq N$ and $A$ bounded above
   then $A$ has the largest element. Then
   same also true if $A \subseteq N$ is replaced by
   $A \subseteq Z$ (why?)

6. If $(\emptyset \neq) A \subseteq Z$ is bounded below
   then $A$ has the smallest element.