Orthogonal projection. Given a nonzero vector \( u \), the unit vector in the direction of \( u \) is
\[
\frac{u}{\|u\|}.
\]
The orthogonal projection of a vector \( v \) along \( u \) is _____________.

Example 1.3. A wooden block of mass \( m \) rests on a plane inclined at an angle of \( \theta \) to horizontal. What forces are acting on the block?

Lines. A line is specified by two points, or equivalently, a point on the line and the direction in which the point can move.

Equation form of a line. In \( \mathbb{R}^2 \), \( ax + by + c = 0 \) represents a line.
Example 1.4. What is the distance between a point \((r, s)\) and the line \(ax + by + c = 0\)?

Parametric form of a line. Given \(a \neq 0\) and \(p \in \mathbb{R}^n\), the points \(\{p + ta : t \in \mathbb{R}\}\) form the line \(L\) through \(p\) in the direction \(a\). (\(t\) is called the parameter.) A point \(x\) is on the line if

\[
\begin{align*}
x_1 &= p_1 + ta_1 \\
x_2 &= p_2 + ta_2 \\
&\vdots \\
x_n &= p_n + ta_n.
\end{align*}
\]

If \(t \in [r, s]\), then \(p + ta\) is the line segment between \(p + ra\) and \(p + sa\). In particular, if \(t \in [0, 1]\), then we are considering the line segment between \(p\) and \(a\).

Exercise 1.5. Find a parametric form of the tangent line to \(x^2 + y^2 = 25\) at \((3, -4)\).

Exercise 1.6. Express the triangle \((0, 0, 0), (1, 1, 1), (2, 0, 2)\) in parametric form. Also find its area.

Symmetric form of a line. For any point \(x = p + ta\) on a line \(L\), we eliminate \(t\) to get the symmetric form of \(L\),

\[
x_j - p_j \frac{a_j}{a_i} = x_i - p_i \frac{a_j}{a_i} \quad \text{if } a_i \neq 0, a_j \neq 0
\]

For example, the line \(2x + y = 3\) has parametric form \(x = 1 + t, y = 1 - 2t\) and symmetric form \(x - 1 = \frac{y - 1}{-2}\).
Example 1.7. Find the equation of the line through $(1, 2, 3, -1)$ and $(-1, -2, 4, -1)$ in 1. parametric form; and 2. symmetric form.

Planes. A plane is uniquely determined by three _____________ points.

Equation form of a plane. In $\mathbb{R}^3$, $ax + by + cz + d = 0$ represents a plane.

Parametric form of a plane. Moving a point in two non-parallel directions yields a plane. Given $\mathbf{p}, \mathbf{a}, \mathbf{b} \in \mathbb{R}^n$, the parametric form of the plane containing $\mathbf{p}$ and spanned by $\mathbf{a}, \mathbf{b}$ is
**Normal form.** In $\mathbb{R}^3$, a normal direction (or simply normal) to a plane is a direction that is orthogonal to the vector formed between any two points on the plane. In general, the plane $ax + by + cz + d = 0$ has normal ________.

In vector terms, given a point $p$ on a plane with normal $n$, the equation of the plane can be written as

**Example 1.8.** 1. Given the plane $ax + by + cz + d = 0$, what is its distance from the origin?

2. Find the distance between $(1, 1, 1)$ and $x + 2y + 3z + 4d = 0$.

**Exercise 1.9.** Find $d$, which is the distance between $2x - 3y + 4z = 7$ and $4x - 6y + 8z = 3$. Find the other plane at a distance $d$ from $2x - 3y + 4z = 7$. 
Angle between planes. What angle does the plane $x + y + z = 1$ make with the $xy$-plane?

Normal and cross product. A normal of a plane containing linearly independent vectors $a, b$ can be obtained by ____________.

Planes in $\mathbb{R}^4$. A plane in $\mathbb{R}^4$ can no longer be specified by one normal direction and a point on the plane. Two normals are needed. The $xy$-plane in $\mathbb{R}^4$ is orthogonal to both $(0, 0, 1, 0)$ and $(0, 0, 0, 1)$. In linear algebra terms, a plane is the ____________ of a 2-dimensional subspace of $\mathbb{R}^4$.

Example 1.10. Given two linearly independent vectors $a, b$ in $\mathbb{R}^4$, the plane through the origin that has normals $a, b$ is given by ____________.

Equation form of a line in $\mathbb{R}^3$. Two planes can ____________, ____________, or intersect at ____________. So a line can also be written as the intersection of two planes.
Example 1.11. Find a parametric form for the line of intersection of $x - y + z = 1$ and $2x + y - z = 0$.

Example 1.12. Show that the line $L$ with parametric form $x = 2 + 3t, y = 1 - 2t, z = -3 - t$ lies on the plane $P : x + y + z = 0$. Find the plane that is orthogonal to $P$ and intersect a plane and intersects $P$ at $L$.

Exercise 1.13. Determine whether the lines $\{(1, 0, -2) + t(-1, 2, 1) : t \in \mathbb{R}\}$ and $\{(0, 2, 1) + t(2, 1, -1) : t \in \mathbb{R}\}$ intersect. What about $x = -1, \frac{y - 3}{-1} = \frac{z - 2}{1}$ and $x - 3 = \frac{y - 4}{2} = \frac{z + 1}{-2}$?

Exercise 1.14. For each of the following quadruple of points, determine they are coplanar, if not, then the distance of $S$ from the plane containing $P, Q, R$:

1. $P = (1, 0, 0), Q = (0, 1, 0), R = (0, 0, 1), S = (1, 1, 1)$;
2. $P = (2, 1, 0), Q = (1, 3, -1), R = (2, 4, -3), S = (1, -1, 3)$;
3. $P = (1, 2, -3), Q = (2, 0, -1), R = (-1, 6, -7), S = (0, 2, 2)$.

Exercise 1.15. Redo the second part of Example 1.12, but the plane has to make an angle $\pi/4$ with $x + y + z = 1$.  

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