Topics: Leibniz's Rule, Some Taylor Polynomial Tricks

1. By Mathematical Induction, show that

$$(f\cdot g)^{(n)} = C_0^n f^{(n)} g^{(0)} + C_1^n f' g^{(n-1)} + \cdots + C_k^n f^{(k)} g^{(n-k)} + \cdots + C_n^n f^{(n-n)} g^{(n)}.$$

(or equivalently, in summation notation, the above is just

$$\sum_{k=0}^{n} C_k^n f^{(k)} g^{(n-k)}.$$

where $f^{(0)} := f$, i.e. the 0th deriviatie of the function f is the function f itself. Also recall that

$$C_k^n = \frac{n!}{k!(n-k)!}$$

2. Find a, b, c, d, e, f in

$$(1 + 2x + x^2)(2 + 3x + x^2 + 5x^3) = a + bx + cx^2 + dx^3 + ex^4 + fx^5$$

3. Following the idea of the preceding question, find the constants p, q, r, s, t, u in

$$(1+x+x^2+x^3+\cdots)(1-x+x^2-x^3+\cdots) = p+qx+rx^2+sx^3+tx^4+ux^5\cdots$$

([1])

[1] The method of finding product of two 'power series' here is useful for solving question 1 in Assignment 4.

Assignments:

1. Compute

$$\int x^5\sqrt{1+x^3}dx$$

2. Compute an approximation to

$$\int_0^1 \frac{\sin(x)}{x}dx$$

with error less than 0.01.

(Hint: You can use the inequality $|\int_a^b f(x)dx| \leq \int_a^b |f(x)|dx$, where $a < b$.)

3. Find $\int_0^{2\pi} x|\cos(x)|dx$

4. Let k be a positive integer, and $a, b > 0$. For any non-negative integer n, define

$$I_n = \int_a^b x^k(\ln(x))^n dx.$$

Show that

$$I_n = \frac{x^{k+1}(\ln(x))^n}{k+1}\Bigg|_{x=a}^{x=b} - \frac{n}{k+1}I_{n-1} \quad \text{for } n \geq 1.$$