Math 3210 Homework 4

Due Date: Nov. 20th, before 18:00. Please put your homework in the assignment box before the due time. The box is on the 2nd floor of Lady Shaw Building opposite to Rm 221. The solution to the homework problems will be uploaded to the course homepage right after the due time. Because of this, *any late homework submission will not be accepted.*

1. Solve the following LPP using big M-method.
Maximize $z = x_1 - 2x_2 - 3x_3 - x_4 - x_5 + 2x_6$ subject to

$$\begin{cases} x_1 + 2x_2 + 2x_3 + x_4 + x_5 & \leq 12, \\ x_1 + 2x_2 + x_3 + x_4 + 2x_5 + x_6 & = 18, \\ 3x_1 + 6x_2 + 2x_3 + x_4 + 3x_5 & \leq 24, \\ x_1, \cdots, x_6 & \geq 0. \end{cases}$$

2. Use big M-method to show that the following LPP does not admit an optimal solution.
maximize $z = 3x_1 + 2x_2$ subject to

$$\begin{cases} 2x_1 + x_2 & \leq 2, \\ 3x_1 + 4x_2 & \geq 12, \\ x_1, x_2 & \geq 0. \end{cases}$$

3. Use simplex method to show the following LPP has infinitely many optimal solutions.
Maximize $z = 2x + 3y$ subject to

$$\begin{cases} x + 3y & \leq 9, \\ 2x + 3y & \leq 12, \\ x, y & \geq 0. \end{cases}$$

4. Find the dual of the following LPP.
Minimize $z = 6x_1 + 6x_2 + 8x_3 + 9x_4$ subject to

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 & \geq 3, \\ 2x_1 + 2x_2 + 4x_3 + 9x_4 & \geq 8, \\ x_1, x_2, x_3, x_4 & \geq 0. \end{cases}$$

5. Find the dual of the LPP:
maximize $z = \mathbf{c}^T\mathbf{x} + \mathbf{d}^T\mathbf{x}'$ subject to

$$\begin{cases} A\mathbf{x} + B\mathbf{x}' & \leq \mathbf{b}, \\ \mathbf{x} & \geq 0, \\ \mathbf{x}' & \text{free}. \end{cases}$$

Here $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{x}' \in \mathbb{R}^m$, so correspondingly, $\mathbf{c} \in \mathbb{R}^n$ and $\mathbf{d} \in \mathbb{R}^m$.

6. Consider the following LPP:
maximize $z = \mathbf{c}^T\mathbf{x}$ subject to

$$\begin{cases} A\mathbf{x} & \leq \mathbf{b}, \\ \mathbf{x} & \geq 0. \end{cases}$$

Suppose we know $\mathbf{b} = (12, 21, 8, 2, 5)^T$. Assume that $\mathbf{u} = (0, 4, 5, 0, 3)^T$ is an optimal solution to the dual of the given problem. Calculate the optimal value of z using strong duality theorem.

7. Consider the following LPP:
minimize $z = y_1 + 2y_2 + y_3$ subject to

$$\begin{cases} x_1 - 2x_2 + x_3 & \geq 2, \\ -x_1 + x_2 + x_3 & \geq 4, \\ 2x_1 + x_3 & \geq 6, \\ x_1 + x_2 + x_3 & \geq 2. \end{cases}$$

Show that $(x_1, x_2, x_3) = (\frac{2}{3}, 0, \frac{14}{3})$ is optimal for this problem, and that $(u_1, u_2, u_3, u_4) = (0, \frac{1}{3}, \frac{2}{3}, 0)$ is optimal for the dual.