MATH 3210 Homework 1

Due Date: Sep. 25th, before 18:00. Please put your homework in the assignment box before the due time. The box is on the 2nd floor of Lady Shaw Building opposite to Rm 221. The solution to the homework problems will be uploaded to the course homepage right after the due time. Because of this, *any late homework submission will not be accepted.*

1. By sketching the feasible set and hyperplane, maximize $z = 3x + 4y$ subject to:

$$\begin{cases} x + 3y & \leq 6, \\ 4x + 3y & \leq 12, \\ x & \geq 0, \\ y & \geq 0. \end{cases}$$

2. Consider the following linear programming problem: find y_1 and y_2 to minimize $y_1 + y_2$ subject to the constraints:

$$\begin{cases} y_1 + 2y_2 & \geq 3, \\ 2y_1 + y_2 & \geq 5, \\ y_2 & \geq 0. \end{cases}$$

Graph the constraint set and solve the problem.

3. Consider the following LPP:
maximize $z = 4x_1 + 2x_2 + 7x_3$ subject to

$$\begin{cases} 2x_1 - x_2 + 4x_3 & \leq 18, \\ 4x_1 + 2x_2 + 5x_3 & \leq 10, \\ x_1 & \geq 0, \\ x_2 & \geq 0, \\ x_3 & \geq 0. \end{cases}$$

First, transform this problem to a standard LPP by introducing additional variables. Then find all the basic solutions of the standard LPP. Finally, solve the LPP by complete enumeration.

4. Show that if the optimal value of the objective function of a LPP is attained at several extreme points, then it is also attained at any convex combination of these extreme points.

5. Let S be the feasible set of a canonical LPP. We convert the LPP to a standard LPP by adding addtional variables. Let S' be its feasible set. Show that every extreme point of S yields an extreme point of S'.

6. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, which means that

$$f(\mathbf{u} + \mathbf{v}) = f(\mathbf{u}) + f(\mathbf{v}) \tag{1}$$

for any $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, and

$$f(r\mathbf{u}) = rf(\mathbf{u}) \tag{2}$$

for any $\mathbf{u} \in \mathbb{R}^n$ and $r \in \mathbb{R}$. Prove that if S is a convex subset of $\mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then $f(S)$ is a convex subset of $\mathbb{R}^m$.

7. Let $A = \begin{bmatrix} 2 & 3 & 4 & 0 & 4 \\ 1 & 0 & 0 & -2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. Determine the existence and degeneracy of the basic solutions with respect to
(a) the first and third colums of A;
(b) the second and last columns of A;
(c) the second and fourth colums of A.