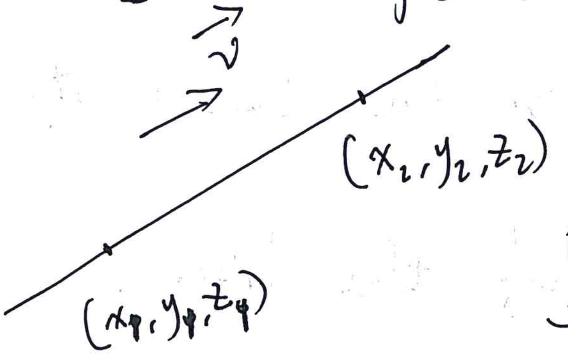
line equation:

how to get the line equation?

case 1: through two points.



idea.

we can get a direction vector of the line.

$$\vec{v} = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$$

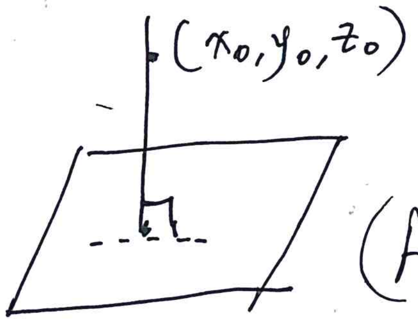
as the line passes through (x_1, y_1, z_1)

So we can set the line equation as

$$\frac{x - x_1}{x_2 - x_1} = \frac{\overset{y}{\cancel{x}} - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Case 2: through one point, perpendicular to a plane.

idea. we can use the general form of the plane equation. set $Ax + By + Cz + D = 0$

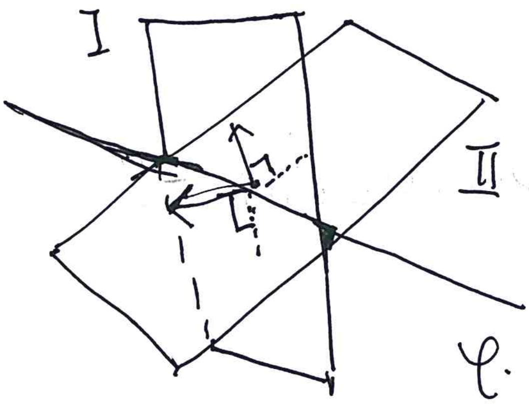


Suppose the line passes through (x_0, y_0, z_0). as. $\vec{v} = (A, B, C)$ is a normal vector of the ~~the~~ plane. So we treat it as the direction vector of the line.

So. we can set the line equation as

$$\frac{x - x_0}{A} = \frac{y - y_0}{B} = \frac{z - z_0}{C}$$

case 3: intersection of two planes.



$$\begin{cases} \mathrm{I}: A_1 x + B_1 y + C_1 z + D_1 = 0 \\ \mathrm{II}: A_2 x + B_2 y + C_2 z + D_2 = 0. \end{cases}$$

ℓ. (A_1, B_1, C_1), (A_2, B_2, C_2) are two normal vector of I, II.

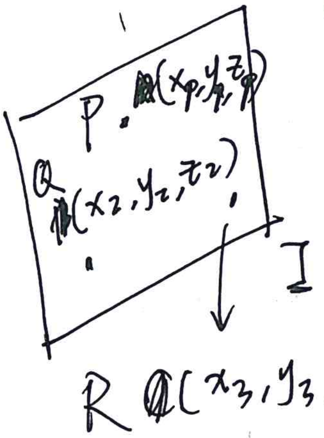
make a cross product $\vec{v} = (A_1, B_1, C_1) \times (A_2, B_2, C_2)$

$:= (v_1, v_2, v_3)$

by the geometric property, we see. $\vec{v}$ can be treated as the directing vector of the line ℓ. meanwhile. we know. the ~~equations~~ number of the equation is 2. ~~but~~ $\mathrm{I} \cap \mathrm{II} \neq \phi$. but the number of variables. is 3. so it has. infinite solutions. we can select any x say $\underline{x_0}$. substitude x_0 in I, II. Combinate I, II and solve corresponding y_0, z_0. then get a point (x_0, y_0, z_0) at last. get the line equation:

$$\frac{x-x_0}{v_1} = \frac{y-y_0}{v_2} = \frac{z-z_0}{v_3}$$

Plane equation: case 1: pass through $\frac{4}{3}$ point



as the general form of the equation is $Ax+By+Cz+D=0$

~~we~~.

idea: we suppose the plane don't pass through the origin. So $D \neq 0$.

so we can change the equation

$Ax+By+Cz+D=0$ to the form

$\frac{A}{D}x+\frac{B}{D}y+\frac{C}{D}z+1=0$. set $A_1=\frac{A}{D}$, $B_1=\frac{B}{D}$

$C_1=\frac{C}{D}$. $\rightarrow$ ⊗ $A_1x+B_1y+C_1z+1=0$.

as point P, Q, R are on I.

So
$$\begin{cases} A_1x_1+B_1y_1+C_1z_1+1=0 \\ A_1x_2+B_1y_2+C_1z_2+1=0 \\ A_1x_3+B_1y_3+C_1z_3+1=0 \end{cases}$$

treat A_1, B_1, C_1. as variables and

Solve the equation. $\Rightarrow$ A_1, B_1, C_1.

method two: $\overrightarrow{PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$\overrightarrow{PR} = (x_3 - x_1, y_3 - y_1, z_3 - z_1)$

make a cross product of $\overrightarrow{PQ}$, $\overrightarrow{PR}$

we can get a normal vector.

$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = (n_1, n_2, n_3)$ ①

after the calculation ①. you can get concrete value.

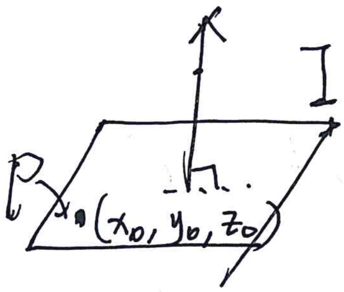
then we can get the plane equation

$n_1 x + n_2 y + n_3 z + D = 0$ ②

as I pt passes through point P, Q, R

we can substitute any point in ② to get D.

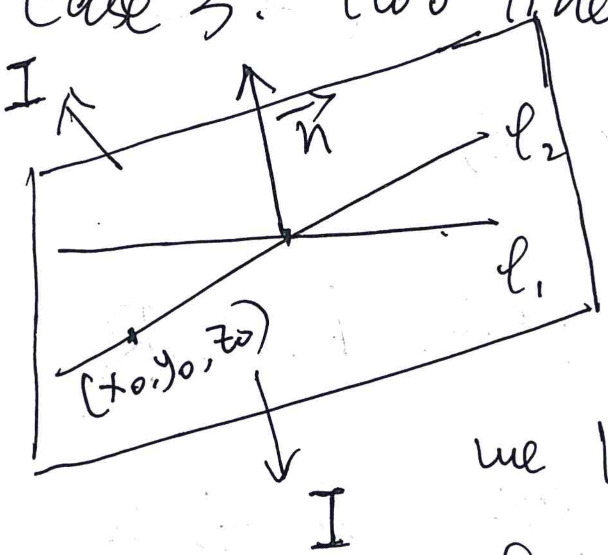
case 2: through one point, a line normal to it.



idea

by using the idea in 1. line equation case. we can get the direction vector of the line $\vec{v} = (v_1, v_2, v_3)$ that's just the normal vector of the plane. we set the plane equation $v_1 x + v_2 y + v_3 z + D = 0$ as (x_0, y_0, z_0) is on I. $\Rightarrow$ D value. then we get the equation.

case 3: two lines that intersect.



l_1, l_2 decide the plane I.

idea: by the method before we know direction vector of l_1, l_2 $\vec{v_1}, \vec{v_2}$ respectively make cross product. $\vec{v_1} \times \vec{v_2}$

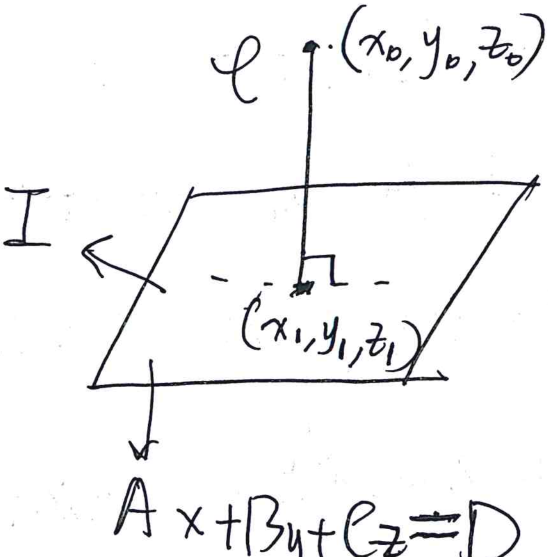
$\Rightarrow \vec{n} = (n_1, n_2, n_3)$.

set $\quad n_1 x + n_2 y + n_3 z + D = 0$. $\cancel{\mathrm{I}}$ ①

Select one point (x_0, y_0, z_0) in ℓ_2 and
Substitute it in $\cancel{\mathrm{I}}$.①. $\Rightarrow D$. then
we have ~~a~~ get the equation for the plane.

Distance problem:

case 1: point and plane.



set (x_1, y_1, z_1) as the graph
the line $\ell \perp \mathrm{I}$ at (x_1, y_1, z_1)
ℓ has the direction vector (A, B, C)
we ~~z~~ can set the line equation

$$\ell: \frac{x - x_1}{A} = \frac{y - y_1}{B} = \frac{z - z_1}{C}$$

as (x_0, y_0, z_0) is on ℓ.

$$\Rightarrow \frac{x_0 - x_1}{A} = \frac{y - y_1}{B} = \frac{z - z_1}{C} \quad ①$$

Set ① $= t$ $\Rightarrow \begin{cases} x_1 = x_0 - At \\ y_1 = y_0 - Bt \\ z_1 = z_0 - Ct \end{cases}$

(x_1, y_1, z_1) on $Ax + By + Cz = D$

$\Rightarrow Ax_1 + By_1 + Cz_1 = D$

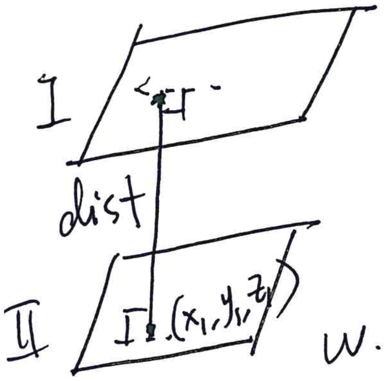
$\Rightarrow A(x_0 - At) + B(y_0 - Bt) + C(z_0 - Ct) = D$

namely $Ax_0 + By_0 + Cz_0 - D = (A^2 + B^2 + C^2)t$ ②

meanwhile, $\text{dist} = \left((x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2 \right)^{\frac{1}{2}}$

$= (A^2 + B^2 + C^2)^{\frac{1}{2}} \cdot |t|$ ③ ~~from~~ substitude t in ②

in ③ $\Rightarrow$ $$\text{dist} = \frac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$$



case 2: two parallelled plane

we can set: $I: Ax+By+Cz=D_1$

w.l.o.g.

$II: Ax+By+Cz=D_2$.

select one point in II.

$$dist = \frac{|Ax_1+By_1+Cz_1-D_1|}{\sqrt{A^2+B^2+C^2}}$$

as (x_1, y_1, z_1) on II. $\Rightarrow Ax_1+By_1+Cz_1=D_2$

$$\Rightarrow dist = \frac{|D_2-D_1|}{\sqrt{A^2+B^2+C^2}}$$